Mathematics and Cosmic Consciousness

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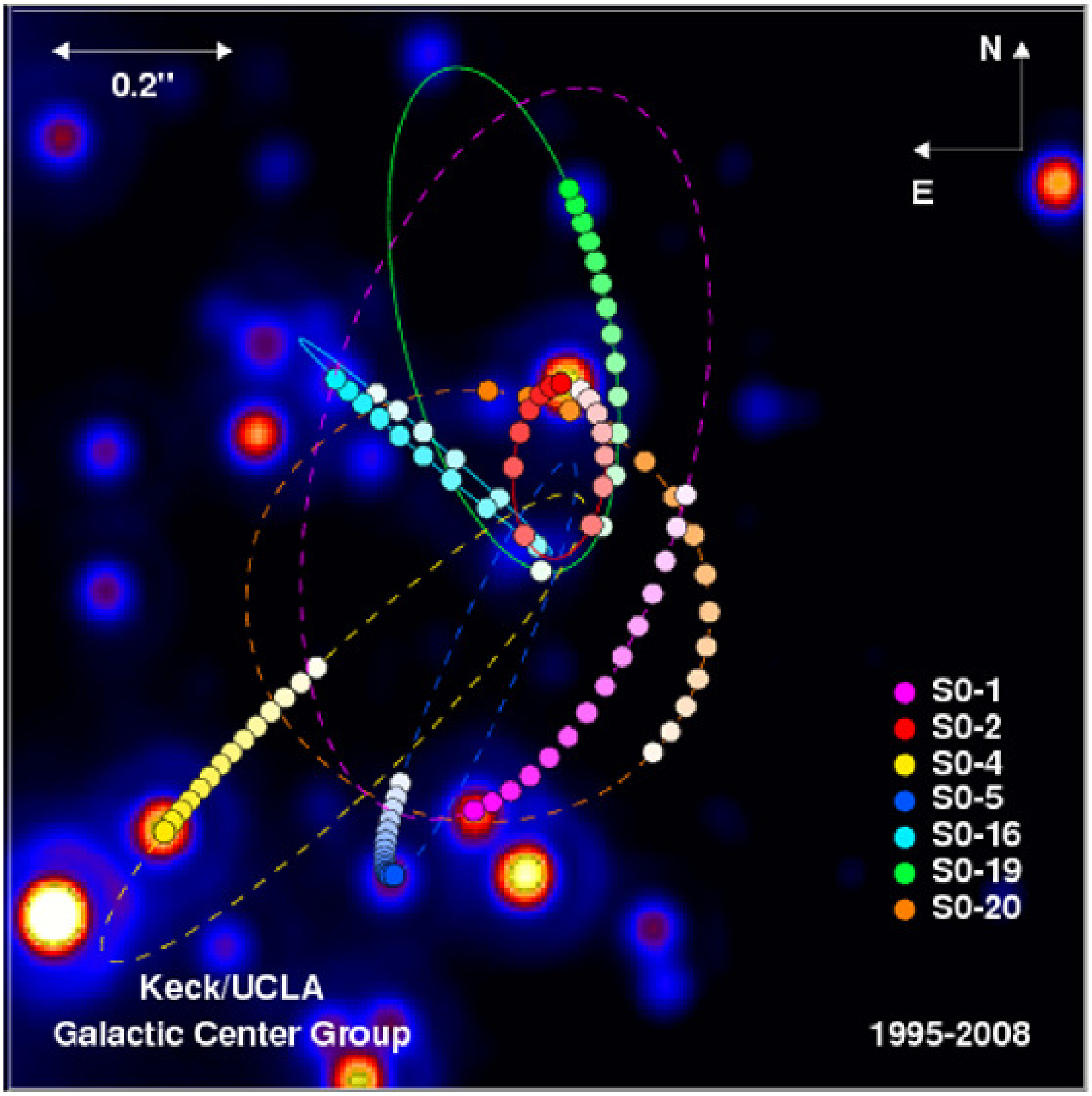


"One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we originally put into them."

Heinrich Hertz, on Maxwell's equations

Ten Cosmic Consciousness Events

310 BC– 230 BC	Aristarchus: "On the Sizes and Distances of the Sun and the Moon"
134 BC	Hipparchus: (Summarized by Ptolemy in Almagest). The precession of the equinoxes (1 degree/ century).
1510	Copernicus: "Commentariolus" ("Little Commentary"). Published mathematical proofs of the heliocentric solar system.
1609	Johannes Kepler: "Asrtronomia Nova". Develops three laws of planetary motion, and Kepler's Equation.

$$\frac{2\pi(t-t_p)}{p} = E - e\cdot\sin(E)$$

1687	Sir Isaac Newton: "Principia". Laws of motion and gravity.

$$F = ma\,, \qquad F = \frac{GMm}{r^2}$$

1817	Friedrich Bessel solves Kepler's Equation.

$$E = \frac{2\pi(t-t_p)}{p} + 2\sum_{n=1}^{\infty}\frac{J_n(n\cdot e)}{n}\sin\left(\frac{2n\pi(t-t_p)}{p}\right)$$

1900	Max Planck. Black body radiation. $E = hf$
1908	Henrietta Leavitt publishes period-luminosity graph of variable stars (Cepheids) in the Small and Large Magellanic Clouds.
1905-1915	Albert Einstein Discovers Special and General Relativity.

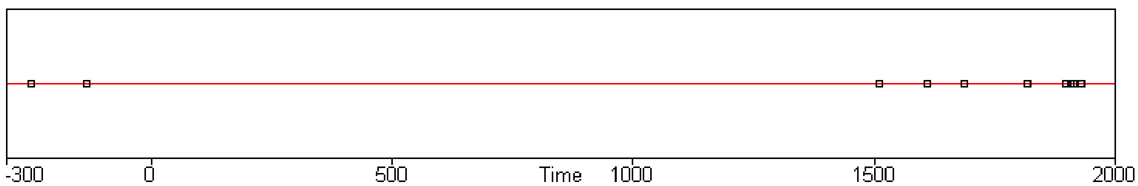
$$R_{uv} - \frac{1}{2}g_{uv}R = -8\pi GT_{uv}$$

1927 Erwin Schrödinger and Paul Dirac.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad \text{Schrödinger}$$

$$-i\hbar c\vec{\alpha}\cdot\vec{\nabla}\Psi(x,t)+\beta mc^2\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad \text{Dirac}$$

Linear placement of the ten events above:



References:

Archives of the Universe, edited by Marcia Bartusiak

The Day We Found the Universe, Marcia Bartusiak

Stars of Heaven, Clifford Pickover.

Archimedes to Hawking, Clifford Pickover.

The Strangest Man, Graham Farmelo.

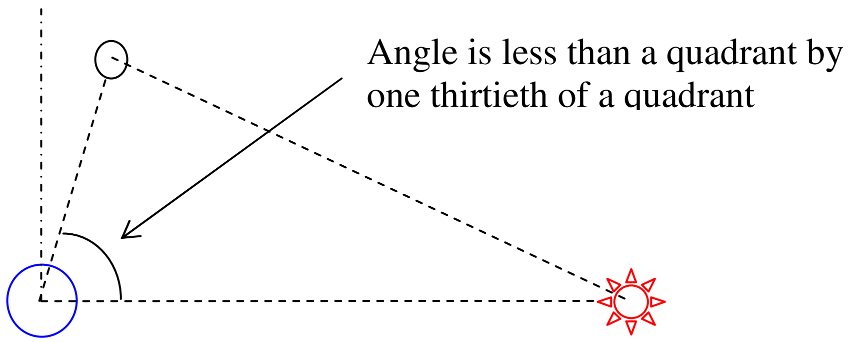
It Must Be Beautiful, Graham Farmelo

Quantum Mechanics, 5th Ed., Alastair I.M. Rae

Quantum Mechanics in a Nutshell, Gerald D. Mahan

One

310 BC – 230 BC Aristarchus: "On the Sizes and Distances of the Sun and the
Moon"



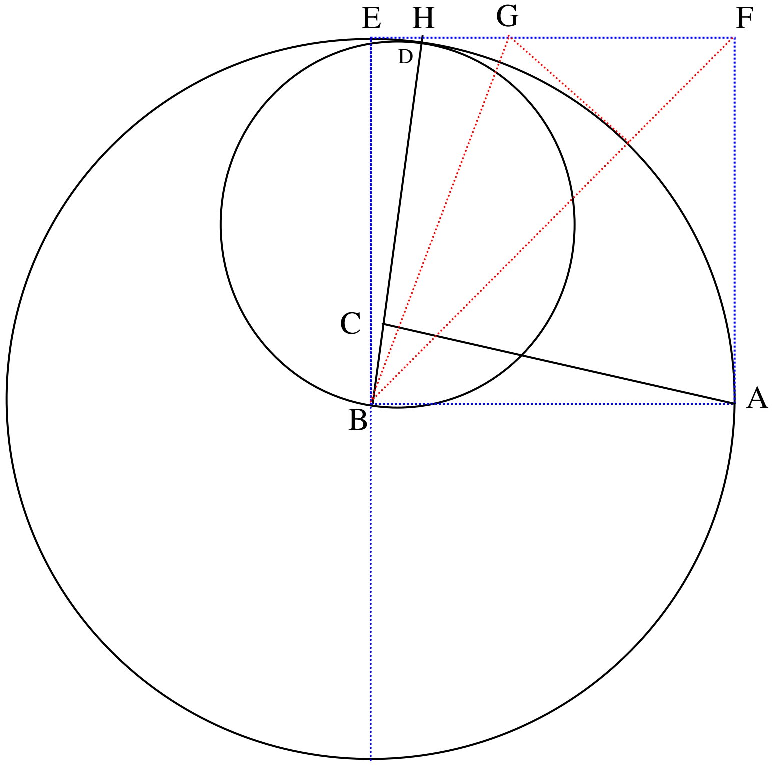
Hypotheses

1.
2.
3.
4. That, when the moon appears to us halved, its distance from the sun is less than that of
   a quadrant by one thirtieth of a quadrant.
5.
6.

Propositions

1.
2.
3.
4.
5.
6.
7. The distance of the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon from the earth.

Proposition 7
(A small sampling of the proof)



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-
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But the angle DBE is also one thirtieth part of a right angle; therefore the ratio of the angle GBE to the angle DEB is that which 15 has to 2 ....

Now, since GE has to HE a ratio greater than that which the angle GBE has to the angle DEB, therefore GE has to HE a ratio greater than that which 15 has to 2.

-
-

$$\frac{GE}{HE} > \frac{15}{2} \qquad (1)$$

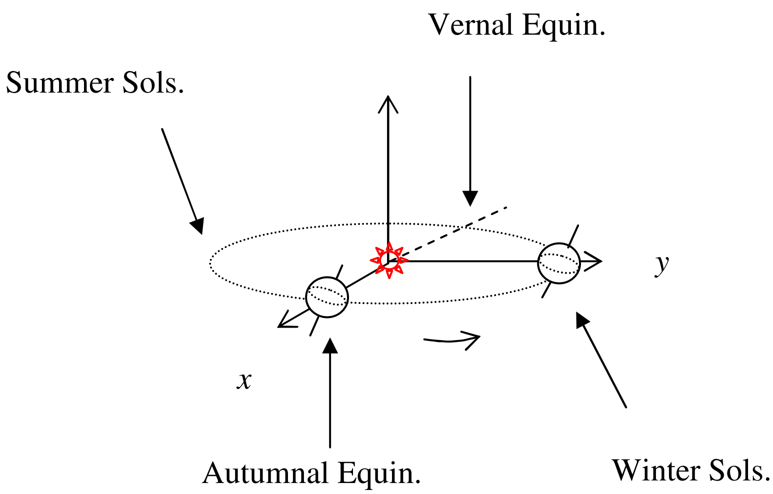
$$\frac{FG^2}{GE^2} = 2, \qquad \frac{FG^2}{GE^2} > \frac{49}{25}, \qquad \frac{FG}{GE} > \frac{7}{5}, \qquad \frac{FG+GE}{GE} > \frac{7+5}{5}, \qquad \frac{FE}{GE} > \frac{12}{5}, \qquad \frac{FE}{GE} > \frac{36}{15} \qquad (2)$$

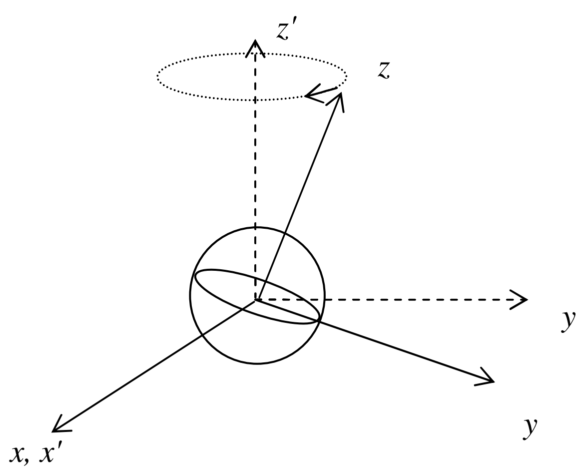
Multiply (1) and (2)

$$\frac{GE}{HE} \cdot \frac{FE}{GE} > \frac{15}{2} \cdot \frac{36}{15}, \qquad \frac{FE}{HE} > 18, \qquad \frac{BE}{HE} > 18, \qquad \frac{BH}{HE} > 18, \qquad \frac{AB}{BC} > 18$$

Two

134 BC Hipparchus: (Summarized by Ptolemy in Almagest) star catalog, the precession of the equinoxes (1 degree/ century).

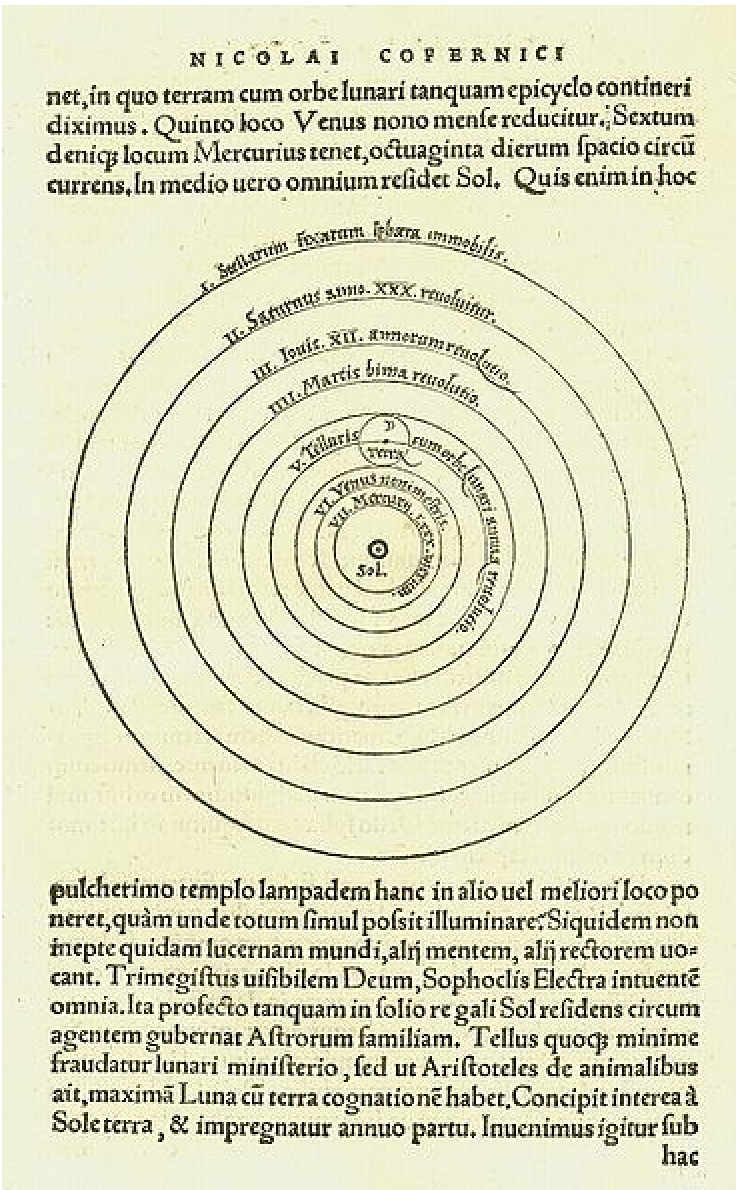




Three

1510 Copernicus: “Commentariolus” (“Little Commentary”). Published mathematical proofs on planetary locations in “On the Revolution of the Heavenly Spheres”, dedicated to Pope Paul III, in 1543, the year of his death.

Explained precession of the equinoxes as 1.4° per century.

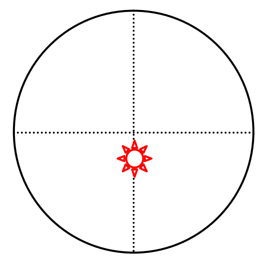


NICOLAI COPERNICI

net, in quo terram cum orbe lunari tanquam epicyclo contineri diximus. Quinto loco Venus nono mense reducitur. Sextum deniq; locum Mercurius tenet, octuaginta dierum spacio circū currens. In medio uero omnium residet Sol. Quis enim in hoc

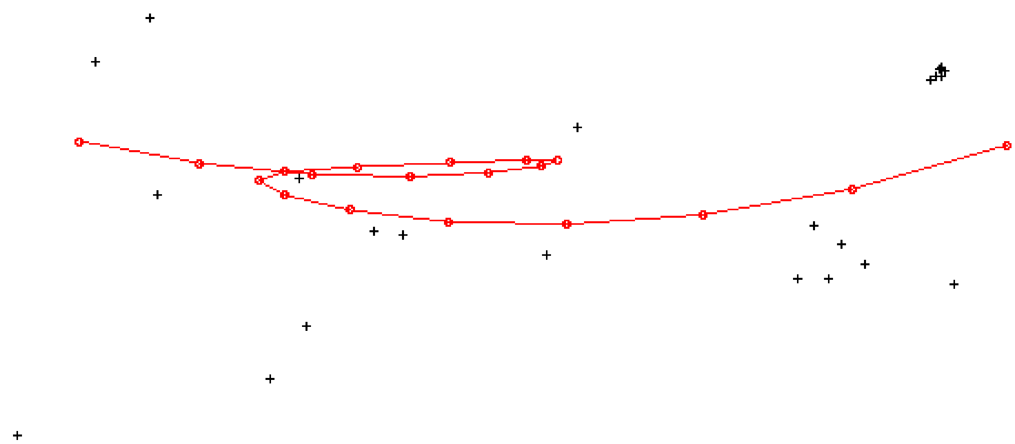
pulcherimo templo lampadem hanc in alio uel meliori loco poneret, quàm unde totum simul possit illuminare? Siquidem non inepte quidam lucernam mundi, alij mentem, alij rectorem uocant. Trimegistus uisibilem Deum, Sophoclis Electra intuentē omnia. Ita profecto tanquam in solio regali Sol residens circum agentem gubernat Astrorum familiam. Tellus quoq; minime fraudatur lunari ministerio, sed ut Aristoteles de animalibus ait, maximā Luna cū terra cognationē habet. Concipit interea à Sole terra, & impregnatur annuo partu. Inuenimus igitur sub hac

The Copernican System

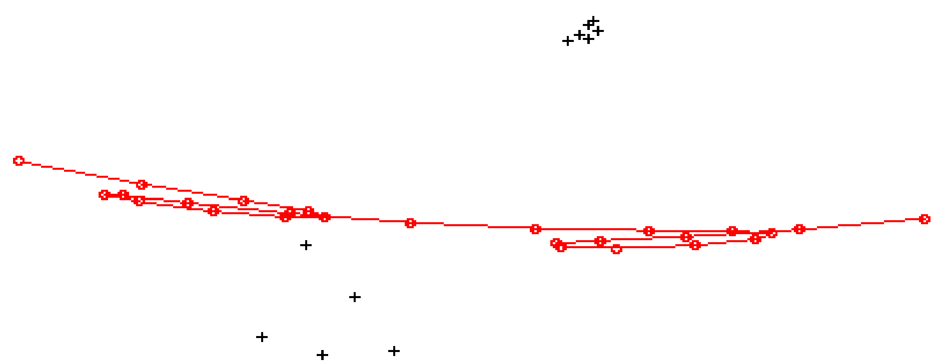


$e = 0.0323$

Copernicus and parallax:



Path of Mars 8/1/07 to 5/1/08 (15 day increments, lat. = 45.5, long. = 120)



Path of Saturn 5/1/00 to 6/1/02 (≅1 month increments, lat. = 45.5, long. = 120)

Four

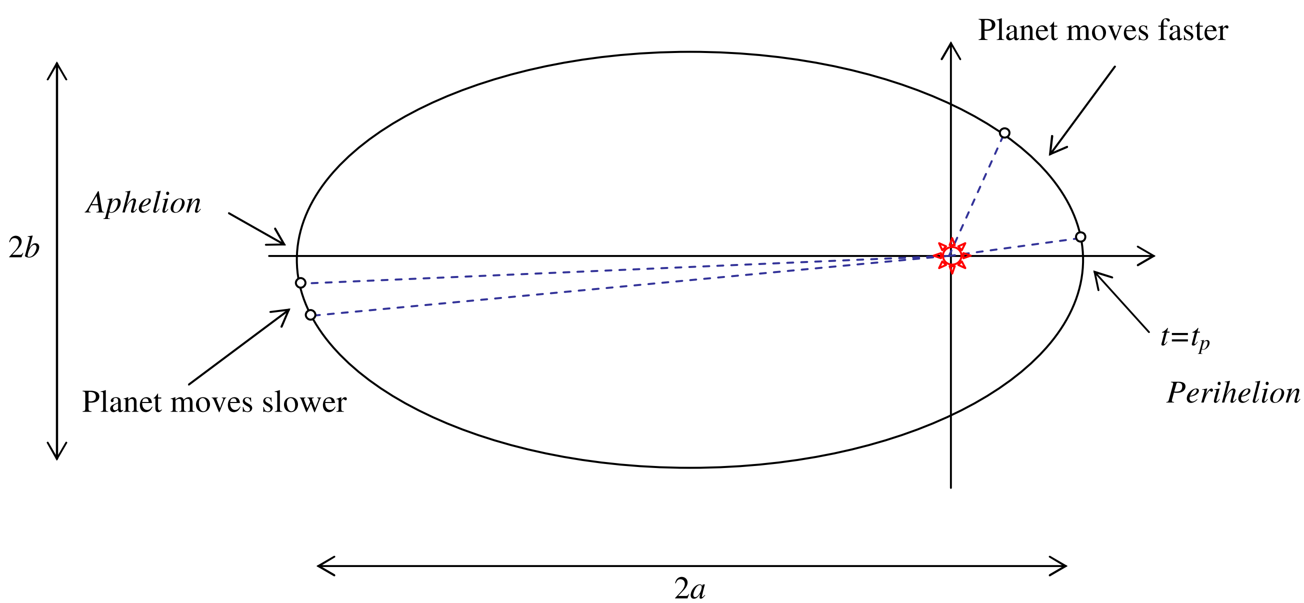
1609 Johannes Kepler: "Asrtronomia Nova". Develops three laws of planetary motion, and Kepler's Equation:

1) A planet orbits the Sun in an elliptic orbit, with the Sun at one focal point.
2) The line joining the Sun to a planet sweeps out equal areas in equal times.
3) The cube of a planet's average distance from the Sun is proportional to the square of its orbital period about the Sun.

$$\frac{2\pi(t-t_p)}{p} = E - e\cdot\sin(E)$$

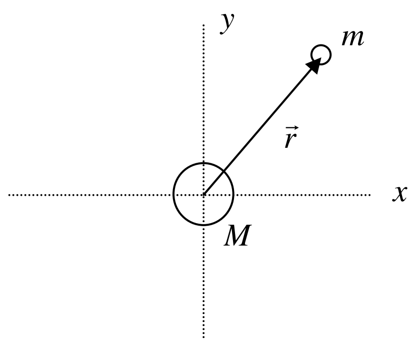
$$\begin{cases} x(t) = a\cos(E) - a\cdot e \\ y(t) = b\sin(E) \end{cases}$$

t is time in years
t_p time at perihelion
p is orbital period in years
e is eccentricity of the orbit
a is semi-major axis in AU
b is semi-minor axis in AU



Five

1687 Sir Isaac Newton: "Principia"



Newton's second law of motion in vector form:

$$\vec{F} = m\vec{a} = m\vec{r}''$$

$$\vec{r} = x\vec{i} + y\vec{j}$$

Newton's law of gravitation in vector form:

$$\vec{F} = -\frac{GMm}{|\vec{r}|^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$

Project idea for Differential Equations:

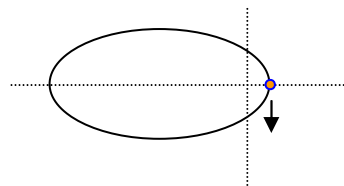
Equate the right hand sides:

$$m(x''\vec{i} + y''\vec{j}) = -\frac{GMm(x\vec{i} + y\vec{j})}{\sqrt{(x^2 + y^2)^3}}$$

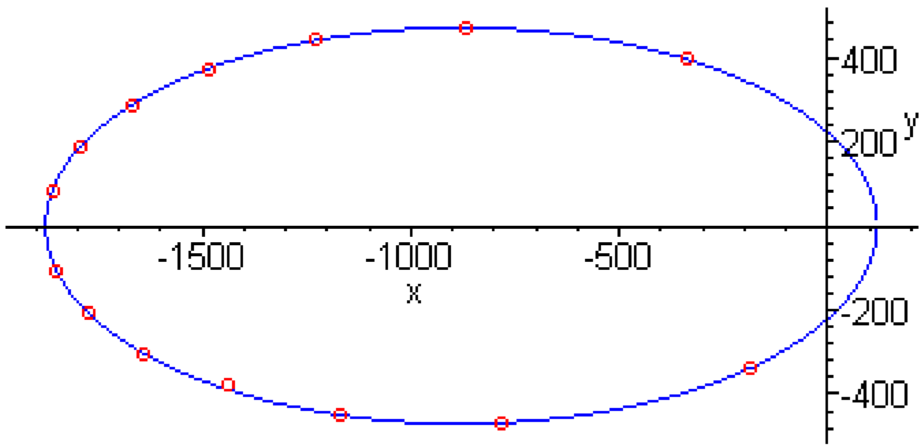
Equate the components:

$$\begin{cases} x'' = -\dfrac{GMx}{\sqrt{(x^2+y^2)^3}} \\ y'' = -\dfrac{GMy}{\sqrt{(x^2+y^2)^3}} \end{cases}$$

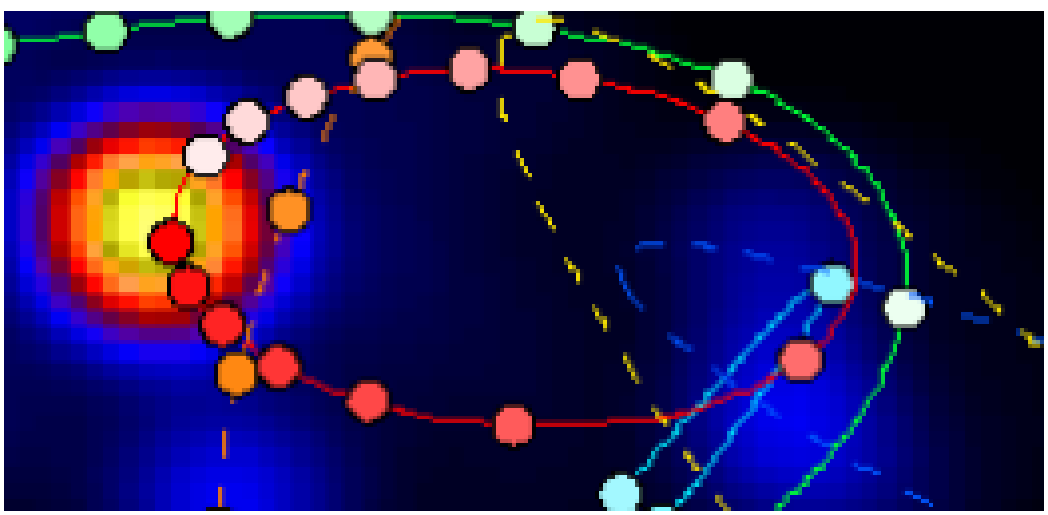
Convert each into two first-order differential equations, include initial conditions for SO2, and solve for 1995 to 2008 with MAPLE:



$$\begin{cases} x' = v_x, & x(0) = 117.7 \quad AU \\ v_x' = -\dfrac{GMx}{\sqrt{(x^2+y^2)^3}}, & v_x(0) = 0 \quad AU/y \\ y' = v_y, & y(0) = 0 \quad AU \\ v_y' = -\dfrac{GMy}{\sqrt{(x^2+y^2)^3}}, & v_y(0) = -1689.7 \quad AU/y \end{cases}$$



400
200
y
-1500
-1000
-500
x
-200
-400

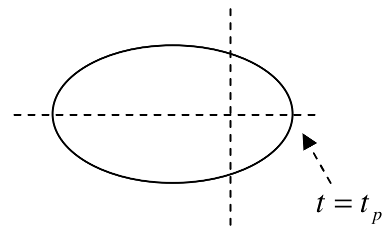


Six

1817 Bessel solves Kepler's Equation.

Kepler's Equation:

$$\frac{2\pi(t-t_p)}{p} = E - e\cdot\sin(E)$$

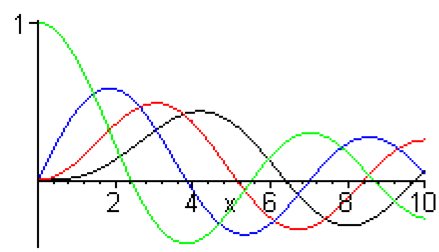


Bessel's solution for E:

$$E = \frac{2\pi(t-t_p)}{p} + 2\sum_{n=1}^{\infty}\frac{J_n(n\cdot e)}{n}\sin\left(\frac{2n\pi(t-t_p)}{p}\right),$$

where $J_n(x)$ is the Bessel function of order *n*:

$$J_n(x) = \sum_{i=1}^{\infty}\frac{(-1)^i (x/2)^{2i+n}}{i!(i+n)!}$$



This gives:

$$E = \frac{2\pi(t-t_p)}{p} + 2\sum_{n=1}^{\infty}\frac{1}{n}\left(\sum_{i=1}^{\infty}\frac{(-1)^i (n\cdot e/2)^{2i+n}}{i!(i+n)!}\right)\sin\left(\frac{2n\pi(t-t_p)}{p}\right)$$

$$\begin{cases} x(t) = a\cos(E) - a\cdot e \\ y(t) = b\sin(E) \end{cases}$$

Expansion for E:

(Let $t_p = 0$)

$$E = \frac{2\pi t}{p} + 2\left[\frac{(e/2)}{0!\cdot 1!} - \frac{(e/2)^3}{1!\cdot 2!} + \frac{(e/2)^5}{2!\cdot 3!} + \cdots\right]\sin(\frac{2\pi t}{p})$$
$$+ \frac{2}{2}\left[\frac{(2e/2)^2}{0!\cdot 2!} - \frac{(2e/2)^4}{1!\cdot 3!} + \frac{(2e/2)^6}{2!\cdot 4!} + \cdots\right]\sin(\frac{4\pi tt}{p})$$
$$+ \frac{2}{3}\left[\frac{(3e/2)^3}{0!\cdot 3!} - \frac{(3e/2)^5}{1!\cdot 4!} + \frac{(3e/2)^7}{2!\cdot 5!} + \cdots\right]\sin(\frac{6\pi t}{p})$$
$$+ \cdots$$

Approximations for E and project ideas for Calculus I, II, III:

$$E \cong \frac{2\pi t}{p} + e \cdot \sin\left(\frac{2\pi t}{p}\right)$$

For sidewalk astronomers

$$E \cong \frac{2\pi t}{p} + e \cdot \sin\left(\frac{2\pi t}{p}\right) + \frac{e^2}{2}\sin\left(\frac{4\pi t}{p}\right)$$

For sunset watchers

$$E \cong \frac{2\pi t}{p} + 2\sum_{n=1}^{10} \frac{J_n(n \cdot e)}{n}\sin\left(\frac{2n\pi t}{p}\right)$$

For semi-serious astronomers

$$E \cong \frac{2\pi t}{p} + 2\sum_{n=1}^{50} \frac{J_n(n \cdot e)}{n}\sin\left(\frac{2n\pi t}{p}\right)$$

For Haley's Comet watchers

$$\begin{cases} x(t) = a\cos(E) - a \cdot e \\ y(t) = b\sin(E) \end{cases}$$

Seven

Max Planck discovers that black bodies radiate in discrete bits:

$$E = hf$$

or

$$E = \frac{hc}{\lambda}$$

$h = 6.626 \times 10^{-34} \quad J \cdot s$ (Planck's constant)

Einstein said radiation itself comes in discrete bits, called them quanta (later photon).

Louis de Broglie discovered that all particles behave like waves, and came up with the de Broglie wave equations for energy and momentum:

$$E = \hbar\omega, \quad \text{or} \quad E = hf$$
$$p = \hbar k$$

$$\hbar = \frac{h}{2\pi}$$

$k = \frac{2\pi}{\lambda}$ wave number

Eight

1908 Henrietta Swan Leavitt publishes period-luminosity graph of variable stars (Cepheids) in the Small and Large Magellanic Clouds.

Henrietta Leavitt (1868-1921) worked at Harvard College Observatory for 30 ¢/ hr.
Worked for Edward Pickering.
Cataloged 1,777 variable stars, 25 of which were Cephieds.

Williamina Fleming and the "women computers"

Henrietta Leavitt

Magelanic Clouds

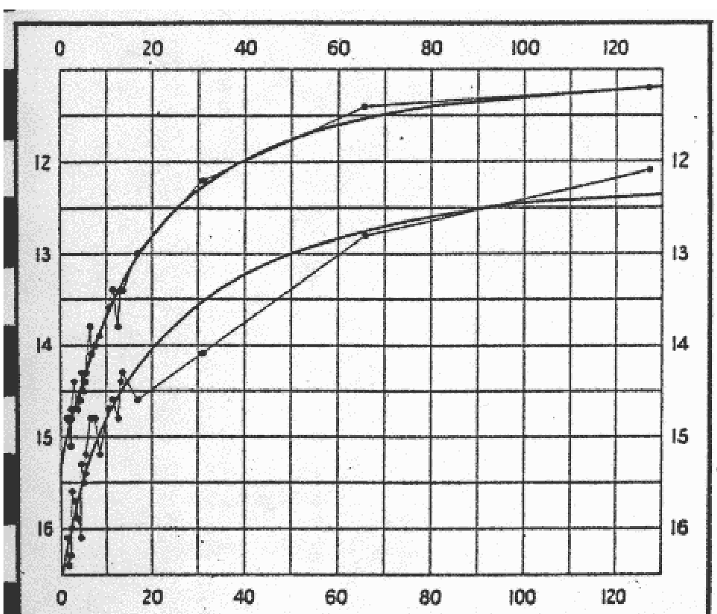


FIG. 1.

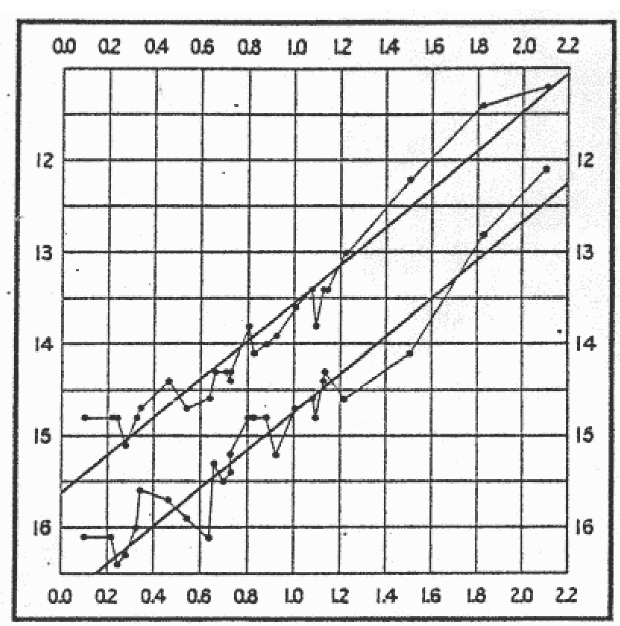
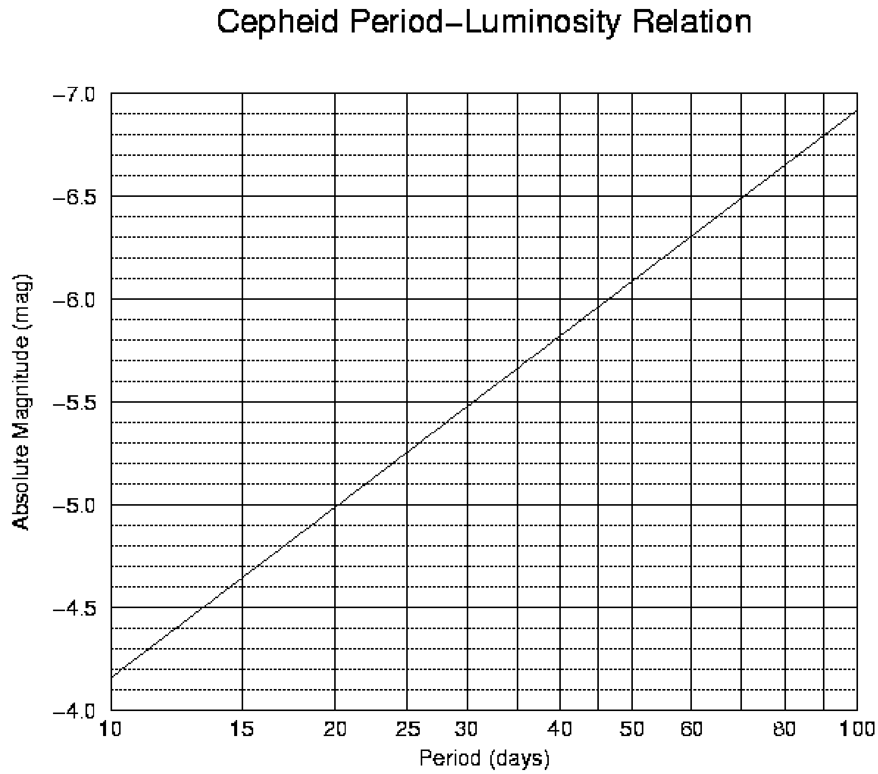


FIG. 2.

Fig. 1 Luminosity vs. period

Fig. 2. Luminosity vs. Log of period.



$$m - M = 5\log\left(\frac{d}{10}\right),$$

m: Apparent magnitude
M: Absolute magnitude
d: Distance in parsecs

Edwin Hubble uses the above equation to fine the distance to Andromeda and Triangulum A Galaxies (1/1/1925, the day we found the universe).

Example: Period is 20 days, apparent magnitude $m = 12.5$
From the graph above, $M = -5.0$

$$12.5 - (-5.0) = 5\log(\frac{d}{10})$$

$$d = 10 \times 10^{(17.5/5)} = 31{,}623 \quad \text{parsecs}$$

$$d \approx 103{,}000 \quad \text{light years}$$

Nine

1905-1915 Albert Einstein: discovers Special and General Relativity.

$$R_{uv} - \frac{1}{2} g_{uv} R = -8\pi G T_{uv}$$

Karl Schwartzchild solves Einstein's general relativity equation in 1916 for a massive star and gets singularities at zero radius and what will become known as the Schwarztchild radius inside the star. Later called the Schwarzchild radius (same as the black hole event horizon). This predicted black holes.

Alexander Alexandrovich Friedmann (Russian) in1922 solves Einstein's equation to predict expanding universe, and backwards in time predicted the Big Bang.

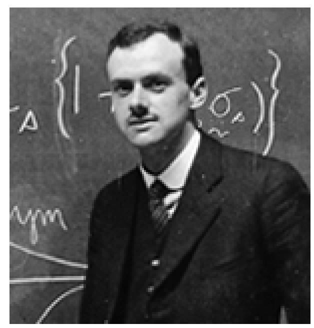
Ten

1927 Erwin Schrödinger, Paul Dirac:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V\Psi(x,t)$$ Schrödinger

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=-i\hbar c\vec{\alpha}\cdot\vec{\nabla}\Psi(x,t)+\beta mc^2\Psi(x,t)$$ Dirac

Schrödinger



Dirac

Traveling waves: $A(x,t)=A_0\cos(kx-\omega t)$

complex form: $A(x,t)=A_0e^{i(kx-\omega t)}$

Classical wave equation:

$$\frac{1}{c^2}\frac{\partial^2A(x,t)}{\partial t^2}=\frac{\partial^2A(x,t)}{\partial x^2}$$

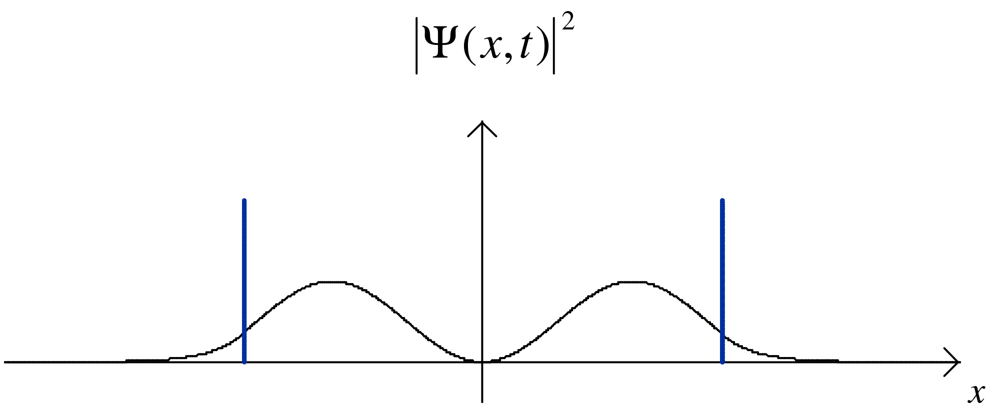
Schrödinger's wave equation:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V \cdot \Psi(x,t)$$

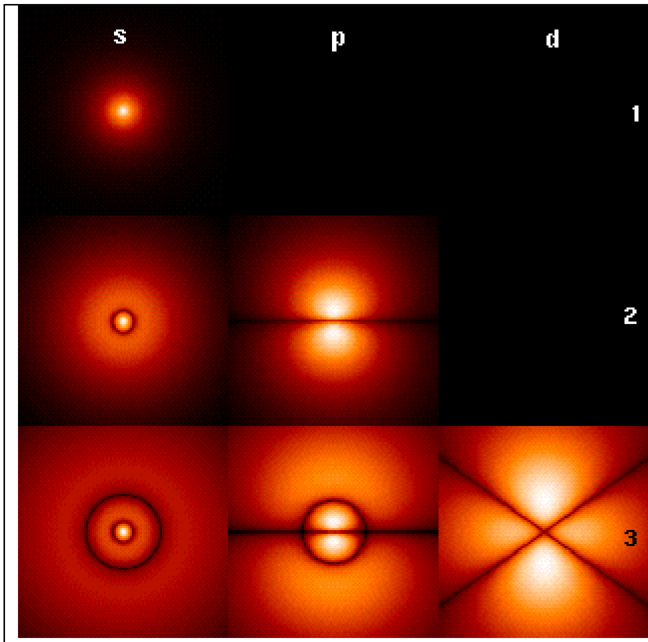
Three dimensional form:

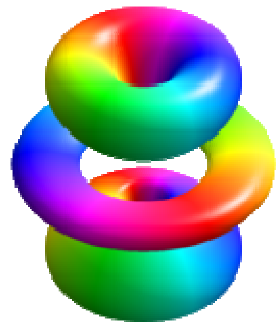
$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi$$

Solution of Schrödinger's wave equation for an electron in a square well (second state):



Solution of Schrödinger's Equation for the hydrogen atom.





Dirac Equation:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -i\hbar c \vec{\alpha} \cdot \vec{\nabla}\Psi(x,t) + \beta mc^2 \Psi(x,t)$$

$$i\hbar \frac{\partial \Psi(x,y,z,t)}{\partial t} = \left[-i\hbar c \alpha_1 \frac{\partial}{\partial x} - i\hbar c \alpha_2 \frac{\partial}{\partial y} - i\hbar c \alpha_3 \frac{\partial}{\partial z} + \beta mc^2 \right] \Psi(x,y,z,t)$$

$$\alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ \alpha_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \ \alpha_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \ \beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\Psi(x,t) = \begin{bmatrix} \Psi_1(x,t) \\ \Psi_2(x,t) \\ \Psi_3(x,t) \\ \Psi_4(x,t) \end{bmatrix} = \begin{bmatrix} \Psi_{e\uparrow}(x,t) \\ \Psi_{e\downarrow}(x,t) \\ \Psi_{p\uparrow}(x,t) \\ \Psi_{p\downarrow}(x,t) \end{bmatrix}$$

The solution to Dirac's equation results in electrons with negative energy. Dirac calls them holes, and later anti-electron, and posits existence anti-matter.

In 1933 anti-electron is discovered (cosmic rays passing through cloud chamber) by Carl Anderson at Caltech.

Appendix A
Short derivation of Schrödinger's Equation

Total energy is kinetic energy plus potential energy:

$$E = \frac{1}{2}mv^2 + V \quad (1)$$

Since momentum p is $p = mv$, equation (1) becomes:

$$E = \frac{1}{2m}p^2 + V \quad (2)$$

de Broglie's equations:

$$E = \hbar\omega$$

$$p = \hbar k$$

Now start with the complex form of the wave equation:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

Take partial with respect to time, and second partial with respect to x:

$$\frac{\partial}{\partial t}\Psi = -i\omega\Psi, \qquad \frac{\partial^2}{\partial x^2}\Psi = -k^2\Psi$$

Compare the above to de Broglie's equations:

$$E\Psi = \hbar\omega\Psi = i\hbar\frac{\partial}{\partial t}\Psi, \quad p^2\Psi = (\hbar k)^2\Psi = -\hbar^2\frac{\partial}{\partial x^2}\Psi \quad (3)$$

Now rewrite Eq. (1) (right multiply by Ψ)

$$E\Psi = \frac{1}{2m}p^2\Psi + V\Psi$$

Plug in from Eq. (3):

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}\Psi + V\Psi$$