The Calculus Concept Inventory

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Introduction!

- What is a Concept Inventory?
- The History of the Calculus Concept Inventory.
- The CCI: Results and Implications.
- Results at OSU.
- What was unique about the sections with higher CCI score?
- Interest in using the CCI?

What is a Concept Inventory.

- A conceptual diagnostic test which aims to assess students' conceptual understanding of key ideas in a discipline, especially those that are prone to misconceptions.
  - They are usually multiple choice.
  - The distracters are designed to elicit misconceptions
  - There is usually no computational component

http://www.flaguide.org/cat/diagnostic/diagnostic2.php

Purposes of Concept Inventories

- To reveal the misconceptions students bring as prior knowledge to a class.
- To measure the conceptual gains of class as a whole.
- To identify concepts that are weak areas of understanding.

http://www.flaguide.org/cat/diagnostic/diagnostic2.php

- Other purposes?

The History of the CCI.

- The Force Concept Inventor – Physics - ~1985

  (Halloun & Hestenes, 1985)

  – Physics, Chemistry, Astronomy, Mathematics
- The Calculus Concept Inventory
  – Patterned after the FCI
  – Developed by a team lead by Jerome “Jerry” Epstein of Polytechnic University (~ 2005)
  – Designed to test the concepts of Differential Calculus
  – 22 multiple choice questions
  – Distracters designed to elicit misconceptions
  – Pretest/Posttest

Scoring of CCI

Normalized Gain of Class:

$$\left[\frac{Gain}{Possible\ Gain}\right]$$

$$\frac{(\text{Class Mean Posttest \%Score}) - (\text{Class Mean Pretest \%Score})}{100 - (\text{Class Mean Pretest \%Score})}$$

Example question similar in format to the CCI

If you know that a function *f(x) is positive everywhere, what can you conclude from that about the derivative f'(x)?:*

a) the derivative is positive everywhere

b) the derivative is increasing everywhere

c) the derivative is concave upward,

d) you can't conclude anything about the derivative

The CCI: Results and Implications.

Epstein & Rhea, draft

Field tested Fall 2006 through 2007

- Field test included many sections of Traditional Lecture (T-L) type classes and a smaller number of Interactive Engagement (I-E) type classes at 15 institutions in the US and one in Finland
- Results of T-L classes:
  - ng between .08 and .23
- Results I-E classes:
  - ng ALL between .30 and .37

Interactive Engagement

- “Interactive Engagement (IE) methods are those designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities which yeild immediate feedback through discussion with peers and/or instructors.”
  - Hake 1998b

University of Michigan Fall 2008

Epstein & Rhea, draft

- Classes
  - 51 sections of Calculus 1
  - Class size < 33 students
  - 1284 of 1342 students completed both pre/post tests
  - HW assignments and exams uniform
  - Harvard Project Calculus Text
  - Instructors attend one week training for I-E and weekly meetings
  - 18 of 51 sections taught by instructors new to the course

- UM results
  - Average n-gain for 51 sections: 0.35
  - Ten sections had gains between 0 .40 and 0.44
  - The range of scores: 0.21 to 0.44
  - Students surveyed about interactivity of class
    - 4-very interactive, 3-interactive, 2 somewhat interactive , and 1-not interactive
    - Average interactivity score: 2.7
    - Range 1.8 to 3.7
    - For 10 sections with gain > 0.4, average interactivity: 3.02
  - Lowest gain section: 0.21; contained 12 high risk for failure students
  - Next lowest section: 0.27

Results at OSU.

- Results from OSU where included in the field test results.
- Total of 13 sections used the CCI

# students	Pretest		Posttest	Abs Gain		N-Gain
– (66)	7.92	-->	9.03	(1.11)	-->	.08
– (10)	10.8	-->	12.00	(1.20)	-->	.11
– (68)	9.03	-->	10.53	(1.50)	-->	.12
– (72)	7.24	-->	9.25	(2.01)	-->	.14
– (72)	7.17	-->	9.18	(2.01)	-->	.14
– (57)	8.05	-->	10.00	(1.95)	-->	.14
– (18)	9.61	-->	11.56	(1.95)	-->	.16
– (45)	5.62	-->	8.42	(2.80)	-->	.17
– (59)	6.71	-->	9.61	(2.90)	-->	.19
– (47)	8.65	-->	11.24	(2.59)	-->	.19
– (54)	8.74	-->	12.44	(3.70)	-->	.28
– (55)	7.07	-->	11.47	(4.40)	-->	.29
– (10)	10.2	-->	15.40	(5.20)	-->	.44

What was unique about the sections with higher CCI score?

- High use of Concept Tests
  - Eric Mazer, Physics, Harvard
- High use of multiple representations, especially visual representations
- Emphasis on meaning and connections
- ???

Concept Test

For Problems 6-7, assume *g(v)* is the fuel efficiency, in miles per gallon, of a car going at a speed of *v* miles per hour.

What are the units of $g'(v)$ =

(a) $(\text{miles})^2$ / (gal)(hour)

(b) hour / gal

(c) gal / hour

(d) (gal)(hour) / $(\text{miles})^2$

Concept Test

Let $N = f(t)$ be the total number of cans of cola Sean has consumed by age t in years. Interpret the following in practical terms, paying close attention to units.

(a) $f(14) = 400$

(b) $f^{-1}(50) = 6$

(c) $f'(12) = 50$

(d) $(f^{-1})'(450) = 1/70$

Exam Question

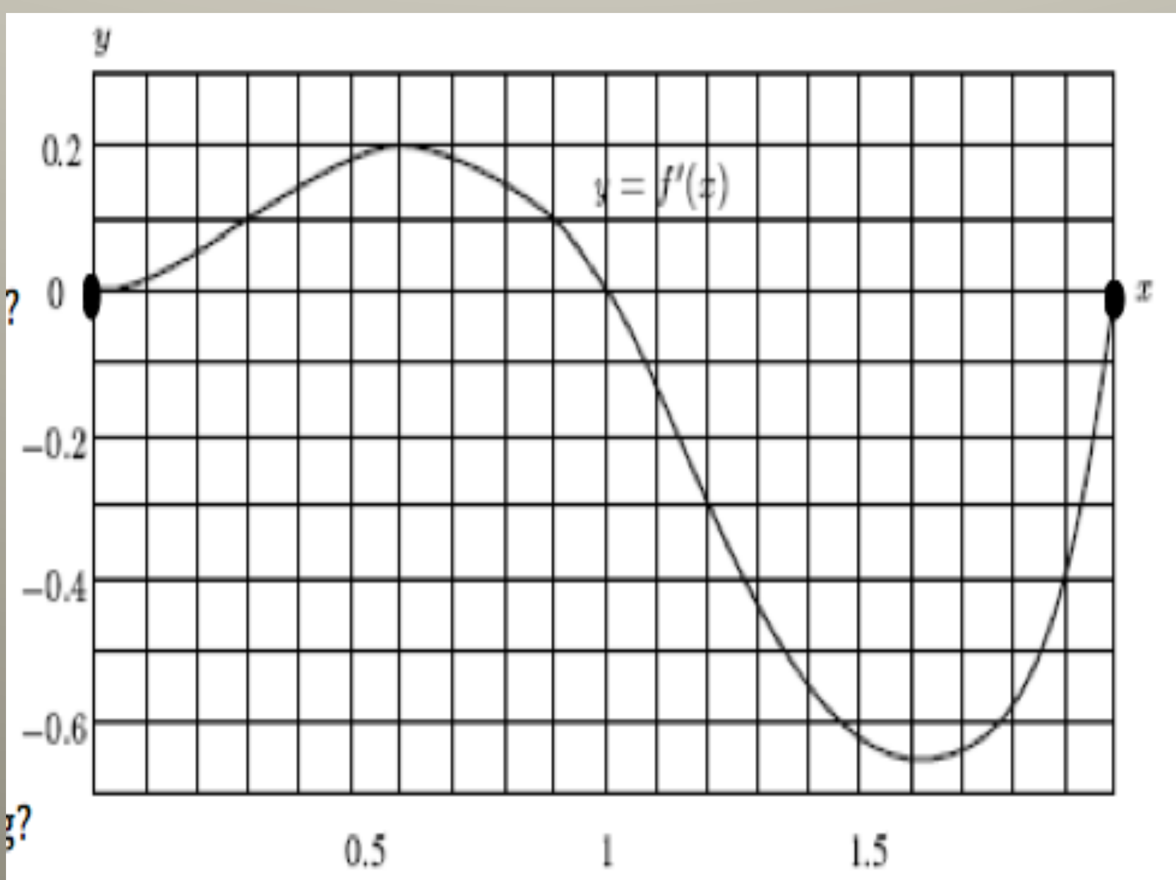
The graph of f ’ is shown below. (This is NOT the graph of f) Use the graph of to answer the following questions. Please put answers in interval notation, to the closest tenth!

(a) On which interval(s), if any, is f increasing?

(b) On which interval(s), if any, is f concave up

(c) On which interval(s), if any, is f“ decreasing?

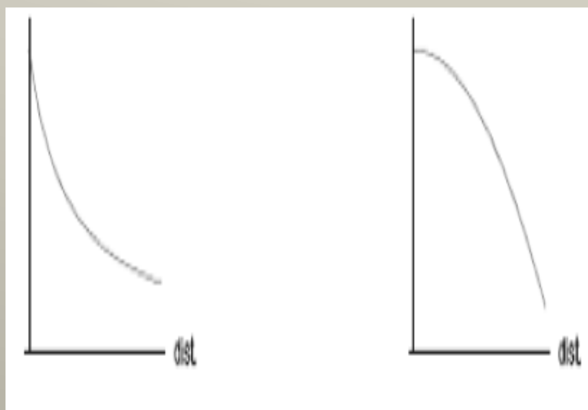
(d) On which interval(s), if any, is f“ positive?



Exam Question

Alone in your dim, unheated room you light one candle rather than curse the darkness. Disgusted by the mess, you walk directly away from the candle, cursing. The temperature (in degrees Fahrenheit) and illumination (in % of one candle power) decrease as your distance (in feet) from the candle increases. In fact, you have a table showing this information! You are cold when the temperature is below 40 . You are in the dark when the illumination is below 50% of one candle power.

a) Two graphs are sketched below. One is temperature as a function of distance and one is illumination as a function of distance. Which is which? (Label the Vertical axis.) Explain!



Distance (feet)	Temp. (°F)
0	55
1	54.5
2	53.5
3	52
4	50
5	47
6	43.5

Distance (feet)	Illumin. (%)
0	100
1	85
2	75
3	67
4	60
5	56
6	53

b) What is the average rate at which the **temperature** is changing when the **illumination** drops from 75% to 56%? (NOTE: Given info about illumination you are being asked about temperature!)

c) You can still read your watch when the illumination is 58%, but not below 58%, so somewhere between 4 and 5 feet. Can you read your watch at 4.5 feet? **Hint:** Think Concavity! **Explain!**

d) Suppose you know that at 6 feet the instantaneous rate of change of the temperature is F/ft and the instantaneous rate of change of illumination is candle-power/ft. Estimate the temperature and the illumination at 7 feet. Explain!

e) Are you in the dark before you are cold, or vice-versa? Explain!

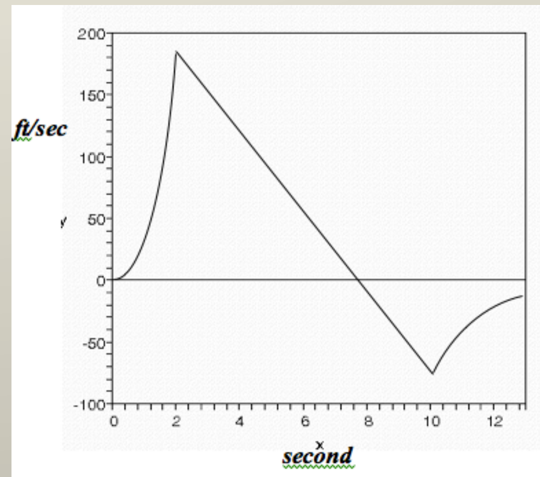
When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket coasts upward for awhile and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts downward. The parachute slows the rocket to keep it from breaking when it lands.

a) Approximately what time did the rocket reach its highest point?

And what was its velocity then?

EXPLAIN t = _______ v =________

This graph shows the **VELOCITY** data from the flight.



b) In approximately what time interval was the acceleration of the rocket constant?

EXPLAIN (,)

c) At what time and approximate how fast was the rocket falling when the parachute popped out? t = _______ v = ________

EXPLAIN

d) At what time and approximate how fast was the rocket was climbing when the engine stopped? t = _______ v = ________

EXPLAIN

Are you interested in using the CCI?

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