A GeoGebra Primer, From Linear Equations to Taylor Functions

GeoGebra is a free software package which does much of the graphing and mathematical computations that a graphing calculator can do. It has some Computer Algebra System commands like Factor[ ] and Expand[ ] and solves equations in one variable. It can also simplify and put basic equations in various forms. But the advantage of GeoGebra is its dynamic nature and ease of use. GeoGebra is available for download at the following website: www.geogebra.org

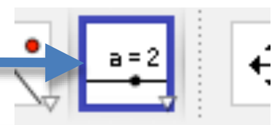
This document gives directions for using GeoGebra for the following general topics:

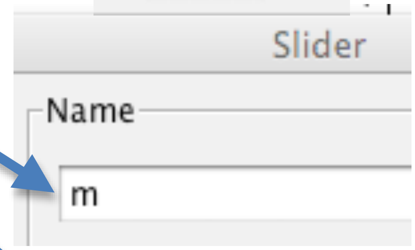
- Demonstrating and exploring functions
- Creating graphs for exams and quizzes
- Generating web-based interactive activities
- Using GeoGebra as a limited replacement for a graphing calculator

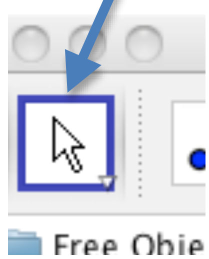
Exploring Functions:

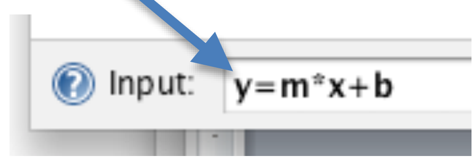
Linear:

1. Open a new GeoGebra page
2. Make a slider named 'm':
    a. Click on the slider icon
    b. Click on the graphing area
    c. Name the slider 'm'
3. Similarly, make a slider named 'b'
4. In the Input Bar, enter y=m*x+b and press return
5. Select the Move icon and adjust the sliders



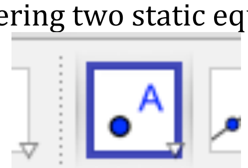


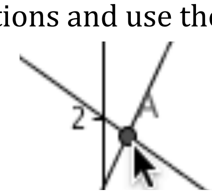




Additional linear explorations:

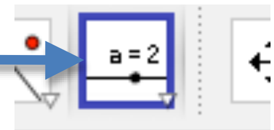
- Use three sliders for other forms of linear equations (y-y1=m*(x-x1) A*x+B*y=C). GeoGebra accepts implicit polynomial equations up to degree 2.
- Trace the equations: Right click the function and select Trace On (or double click and open Object Properties to change the color, change thickness, etc.)
- Get Values from the equation by putting a point on the equation: Select the New Point icon and click on the line. The point values will display in the Algebra Window. Rounding can be altered under the Options menu.
- Solve a system of equations by entering two static equations and use the New Point icon to place an intersection:

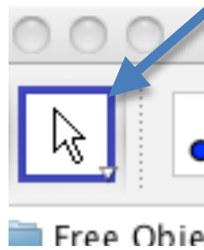


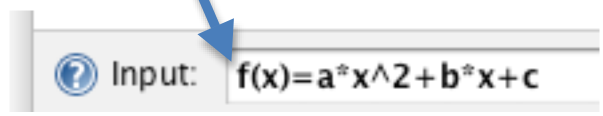


Quadratic:

1. Open a new GeoGebra page
2. Make a slider named a:
    a. Click on the slider icon
    b. Click on the graphing area
3. Similarly, make sliders named b and c
4. In the Input Bar, enter f(x)=a*x^2+b*x+c and press return
5. Select the Move icon and adjust the sliders

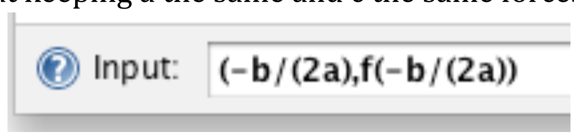






Additional quadratic explorations:

- Examine the effect from b: In the Input Bar, create the parabola's vertex: (-b/(2a),f(-b/2a)), Right click to select Trace On for the vertex point, and alter b. It is interesting to note that keeping a the same and c the same forces an identical parabolic path for b.



- Use sliders for other forms: y-y1=a*(x-x1)^2 or a*(x-p)(x-q). GeoGebra defaults to function notation for expressions in terms of x.
- Trace the equations: Right click the function and select Trace On (or double click and open Object Properties to change the color, change thickness, etc.). Using arrow keys instead of the mouse will trace all possible slider values and creates quite a beautiful display.

Values of base, b in a*b^x

- Make sliders for a and b.
- Examine what happens to the graph when b is 0 and when b is negative.

For an example of a polished version of this activity, which can be used as an at-home assignment see my following applet which is more polished:

spot.pcc.edu/~jpettit/exponential/Continuous_Exponential_Assignment.html

Translations:

- Open a new GeoGebra Page
- In the Input Bar, type x^2 return
- Use the mouse to click and hold the parabola and translate it. Note the change in the symbolic function in the Algebra Window.
- Enter f(2x) in the Input Bar and move the original parabola again
- Enter 2f(x) in the Input Bar and move the original parabola again
- Alternately, for calculus 1, enter f’(x) and f’’(x) and move the parabola to examine the effect on the first and second derivative.

Polar coordinates: To plot a polar function, use the parametric equation command: Curve[cos(t)*f(t), sin(t)*f(t),t,0,4pi] for two rotations of function f, starting at 0.

Statistics (best fit line):

- Use the New Point tool to make a handful of generally linear points on the screen.

- Use the Best Fit Line icon (under the Perpendicular Line icon) or enter fitline[{ and the names of the points separated by commas. Then close the brace and bracket: }] Note: Sets of points can also be entered by holding the option key down and highlighting an area of the graph.

- Use the Move tool to change one point to an outlier. The effect is quite dramatic

- Find the correlation coeffecient using the command CorrelationCoeffecient[ ] with the data points in a bracketed set, separated by commas.

Statistics (z-score mapping animated):

- Create the standard normal curve by entering the following into the Input Bar:1/sqrt(2pi)*e^(-x^2/2)

- Use the Move Graphics View icon (or the zoom options under the Move Graphics View icon). Alternately, you can use the roller bar on the mouse to zoom in and out, or right click on the background to change the screen graph directly.

- Create sliders mu and sigma

- In the Input Bar, enter the following:

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1 / sqrt(2 π*σ^2) e^(-(x-μ)^2 / 2)
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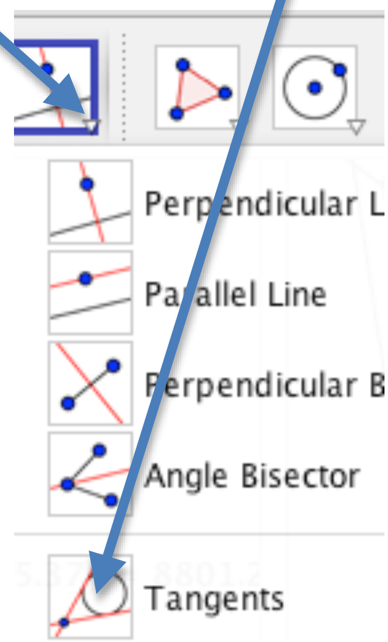
Calculus:

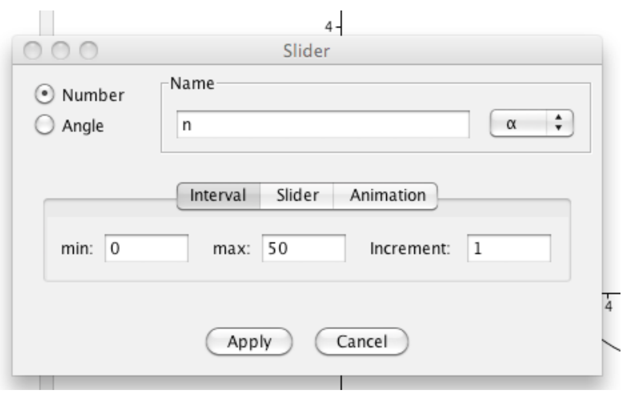
- Secant lines: Enter an equation; Use the New Point tool to put two points on the function. Connect them using the Line Through Two Points tool.

- Tangents: Use the New Point tool to put a point on a function. Use Tangent tool under the pull down triangle on the Perpendicular Line icon then click on the point and function.



- Derivative notation: f'(x), f''(x), etc. give derivatives.
- Riemann sums: See Tom Getty's link below.
- Taylor series: Enter a function, say f(x)=sin(x)

  Create a slider, n that goes from 0-50 incrementing by 1:



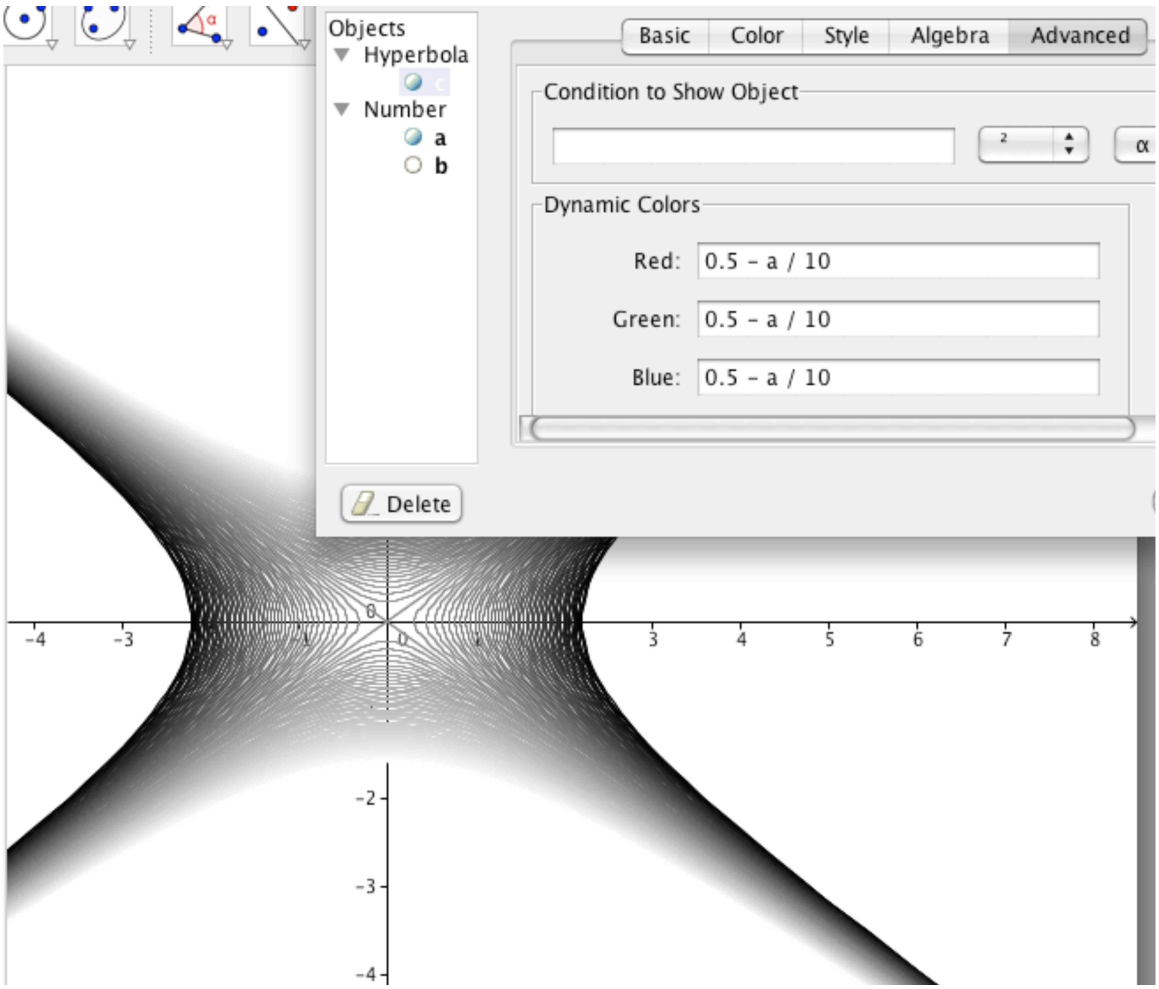
  Create a slider, 'a' for the x-value, and enter the point (a,f(a)) in the Input Bar

  In the Input Bar, enter TaylorPolynomial[f(x),a,n].

  Tracing this function is quite beautiful.

- 3D traces. Although Geogebra does not support 3D graphing, following are directions on contour tracing using z=x^2-2y^2

  Make a slider a. In the Input Bar, enter a=x^2-2y^2 Right click the equation, go to Object Properties and click on the Advanced tab. The dynamic colors vary in hue between each integer, including negatives. Entering 0.5-a/10 for each of the three colors will give a depth to the function of z(x,y) as 'a' varies from -5 to 5, looking as if the solid is fading into a mist:



  Leaving Red and Green blank, and incrementing the slider by 1, gives a more traditional trace of the function.

- Mapping (u,v) onto (f(u,v),g(u,v)): For example -- make sliders u and v, each with min 0 and max 2; also, when making u and v, click on Animation, and set the speed of u to 5 and v to 7 (in order to cover the domain). For each slider, right-click them and select Animation On. In the input Bar, make point (u,v).

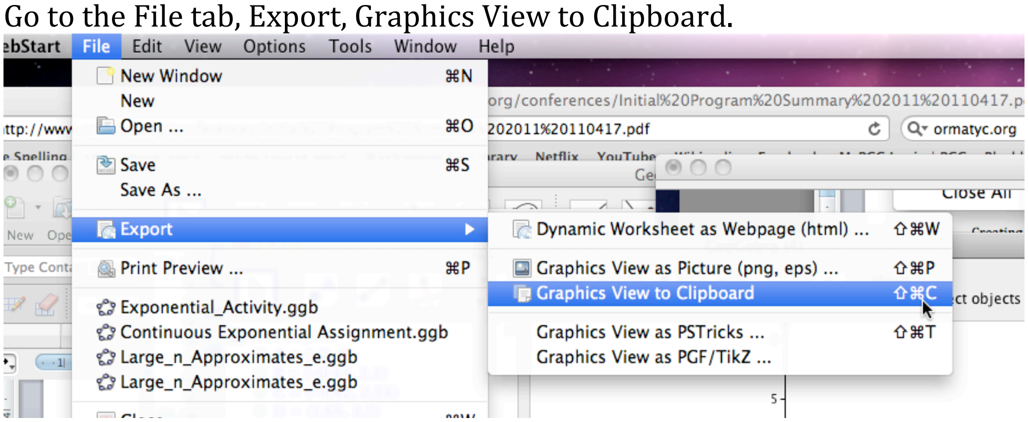
- Similarly, make point (f(u,v),g(u,v)), for this example (-u*v,u-v). Setting them to different colors in Object Properties helps distinguish them.

Creating Graphs for tests, exams and quizzes

As an example, following is how to create and copy/paste the graph of the piecewise function into a document (e.g. Microsoft Word):

$$f(x) = \begin{cases} 2 - x & -2 \le x < 1 \\ 3 & 1 \le x \le 5 \end{cases}$$

- Create the endpoints of each line segment using the New Point tool, or enter the points in the Input Bar: (-2,4) enter, (1,1) enter, etc.
- Right-click one of the point (1,1) and open the Object Properties. Click on the Style tab. Change the Point Style to an empty point. If desired, change the size and labeling by clicking on Point to select all the points and then adjust them as desired.
- Use the Line Segment tool or type segment[A,B] etc to connect the points.
- To add a background grid, change axis scales, background colors etc, right click in the Graphics Window and adjust as needed.
- To export this picture, resize the window so it shows what you want to paste. Go to the File tab, Export, Graphics View to Clipboard.



- On your document, paste the picture from the clipboard (i.e. control V)

To make graphs of polynomials (or other common functions):

- Create several points.
- In the Input Bar, enter FitPoly[{A,B,...}, degree of desired polynomial]. The best-fit polynomial will be formed based on those points. Move the points to adjust the polynomial.
- When finished, hide the points by clicking on the blue circles in the Algebra View Screen and export it to the clipboard as mentioned above.

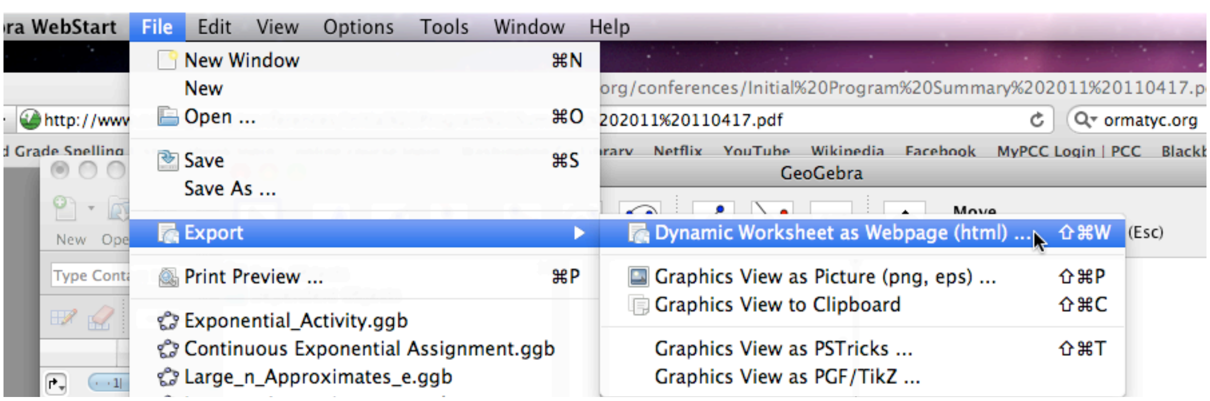
To make many objects, use the Spreadsheet, which works similarly to Excel. Any object you make in the Spreadsheet appears in Graphics view. Values and equations are not objects. Following is how to make the table and graph of an interest function with 12% interest compounded annually.

- Under the menu tab View, select the Spreadsheet View.
- In cells A1 and A2, enter 1 and 2. Highlight them and use the handle in the bottom right corner to pull down to row 10, making a list of integers 1-10.
- In cell B1 enter 1.12. In cell B2, enter 1.12*B1. Highlight cell B2 and use the handle to pull the rectangle down to B10. (Alternately, enter 1.12^A1 in B1 and highlight it and pull it down to B10)
- In cell C1 enter (A1, B1). In cell D1 enter (A2,B1). In cell E1 type segment[C1,D1]. Two points and a segment will be created on the Graphics Screen.
- Highlight Cells C1, D1 and E1 at the same time and pull their handle down.
- To create holes, Go to Object Properties by right-clicking on a point. Highlight all the D-column points and change their style to empty hole.

Creating activities

One of the best features of GeoGebra is the option to export constructions as web pages. With a great amount of control on what features users can and cannot access, instructors can create assignments and instructional support material. Following are instructions on exporting GeoGebra files as .html files and details on features that make the process simpler.

- Once a construction is ready for export as a webpage, select the File pull-down menu, Export, Dynamic Worksheet as Webpage (html).



- You can title the web applet, and include your name as author. The text before and after the webpage may or may not display on the final product depending on the browser used and the window size the web page opens in. In general, to guarantee text is included on the webpage, include it as text on the Graphics screen directly.

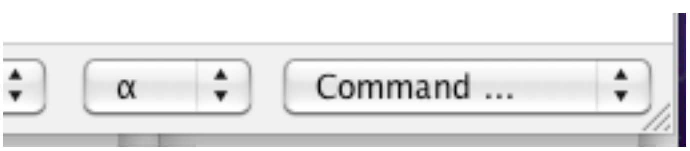
- The Advanced tab allows you to control window size, user access to tool icons, user ability to alter the line stiles and colors, etc.

- Important details include the fact that users need Java installed on the computer they are running the applet on. Also, the .jar files that are created upon export need to be in the same folder as the .html file(s). Loading the file and running the applet sometimes takes a few seconds.

Choosing simplicity often better ensures success. Offering an activity with one or two sliders instead of three or four avoids confusion for students unfamiliar with computer activities. Objects can be affixed in Object Properties; it is useful to "fix" sliders, for example, so users don't move the slider location when trying to alter the slider value. Construction objects (that is, objects used to construct other objects but need not be seen) can be hidden *en masse* in Object Properties or using the Show / Hide Object icon under the Move Graphics View icon. To give trace options, make two identical functions, trace one, but make it shown optionally using a Check Box to Show / Hide Object (available under the slider icon). Objects can be hidden under specific circumstances using the Advanced tab in the Object Properties; in other words, using sliders with >, <=, etc. (Boolean operators are available for conditions such as "not", "and", "or", etc.) Images can be imported and anchored to one, two or three points, or no points at all.

Use as a graphing calculator:

The Command tab shows the breath of operations that GeoGebra can do.



Not the least of which is the regression capability using the myriad of Fit commands, a much simpler and intuitive interface compared to graphing calculators. Entering an equation up to degree 2 in one variable in the Input Bar will provide numerical answers (and graphical objects). Right clicking on points gives the option for polar coordinates and right-clicking linear equations in the Algebra window gives the option for slope-intercept or standard form. The Extremum[ ] command works for polynomials and finds all points simultaneously. The Spreadsheet view works well for tables, and accepts copied data from Excel. In the Input Bar, entering numbers with x= and y= creates vertical and horizontal lines. The Intersecting Objects icon can be used to find all intersection points for polynomials simultaneously.

There is a great deal to learn about GeoGebra, and I hope instructors and students alike find new ways of understanding mathematical truths through its use. Feel free to contact me with any questions, corrections or comments: jeffrey.pettit@pcc.edu

Tom Gettys from Lane Community College in Eugene, Oregon offered his extensive library of self-written GeoGebra demos and activities. I encourage you to visit his impressive list:

http://media.lanecc.edu/users/gettyst/GeoGebraDemonstrations.html

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