Calculus 2 Nirvana

Dr. William T. Webber
Whatcom Community College

Washington State Community College
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Nirvana (from dictionary.com)

1. Buddhism: The ineffable ultimate in which one has attained disinterested wisdom and compassion.
2. Hinduism: Emancipation from ignorance and the extinction of all attachment.

Nirvana: (from WTW)

1. The ultimate state of awareness.
2. Enlightenment attained from an extended journey.

Recap of Calculus 1 Nirvana

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$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Genesis of this talk

- Achieved Calculus 1 Nirvana
- Attendee asked "Is there a Calculus 2 Nirvana?"
- Gave Talk in Spring 2010
- Taught Calculus 3 in Fall 2010
- Students achieved Calculus 1 Nirvana
- Later in Quarter student asked "Is this Calculus 2 Nirvana?"

A Journey to Enlightenment

- Discovery
- Investigation
- The Slog
- Enlightenment

Discovery

- Integrals by Riemann sums
- Integrals by Fundamental Theorem
- Applications of Integrals
  - Volumes of Revolution by washers
  - Volumes of Revolution by shells
  - Average Value of a Function
  - Center of Mass
  - Papus' Theorem

Investigation

- Volume of revolution by washers

Investigation

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- Region in xy-plane bounded on left by $x = a$, on right by $x = b$, above by $y = f(x)$ and below by $y = g(x)$.

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- Region in xy-plane bounded on left by $x = a$, on right by $x = b$, above by $y = f(x)$ and below by $y = g(x)$.
- Rotate around x-axis.

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- Chop original region into vertical rectangles

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- Volume of washer is $\pi(f^2(x) - g^2(x))$

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- Generate little bits of volume that look like washers.
- Volume of washer is $\pi(f^2(x) - g^2(x))\,dx$
- Take integral to add up all the little bits.

Investigation

- Volume of revolution by washers
- Region in xy-plane bounded on left by $x = a$, on right by $x = b$, above by $y = f(x)$ and below by $y = g(x)$.
- Rotate around x-axis.
- Chop original region into vertical rectangles
- Generate little bits of volume that look like washers.
- Volume of washer is $\pi(f^2(x) - g^2(x))\,dx$
- Take integral to add up all the little bits.

$$V = \pi \int_a^b f^2(x) - g^2(x)\,dx$$

Investigation

- Volume of revolution by shells
- Region in xy-plane bounded on left by $x = a$, on right by $x = b$, above by $y = f(x)$ and below by $y = g(x)$.
- Rotate around y-axis.
- Chop original region into vertical rectangles
- Generate little bits of volume that look like shells.
- Volume of washer is $2\pi x(f(x) - g(x))\, dx$
- Take integral to add up all the little bits.

$$V = 2\pi \int_a^b x(f(x) - g(x))\, dx$$

Investigation

- Average Value of a Function
- Interval from a to b.
- Integrate $f(x)$ from a to b.
- Divide by the length of the interval
- Sometimes I do this by chopping up.

$$Avg = \frac{\int_a^b f(x)\,dx}{b-a}$$

Investigation

- Center of Mass
- Chop region into vertical rectangles
- Calculate $\bar{x}$
  - Multiply area of rectangle by distance from y-axis
  - Integrate to add up all the bits of moment.
  - Divide by area of region.

$$\bar{x} = \frac{\int_a^b x(f(x) - g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

Investigation

- Center of Mass
- Chop region into vertical rectangles
- Calculate $\bar{y}$
  - Multiply area of rectangle by distance from x-axis
  - Take the average distance. $(f(x) + g(x))/2$.
  - Integrate to add up all the bits of moment.
  - Divide by area of region.

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f(x) + g(x))(f(x) - g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

Investigation

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f(x)+g(x))(f(x)-g(x))\,dx}{\int_a^b (f(x)-g(x))\,dx}$$

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f^2(x)-g^2(x))\,dx}{\int_a^b (f(x)-g(x))\,dx}$$

Investigation

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- Volume of Revolution

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Investigation

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- Volume of Revolution
- Region of area A is rotated around an axis.
- Volume generated is $V = 2\pi r A$
- r is distance from axis to centroid of region.

Investigation

$$V = \pi \int_a^b f^2(x) - g^2(x)\, dx$$

$$V = 2\pi \int_a^b x(f(x) - g(x))\, dx$$

$$Avg = \frac{\int_a^b f(x)\, dx}{b - a}$$

$$\bar{x} = \frac{\int_a^b x(f(x) - g(x))\, dx}{\int_a^b (f(x) - g(x))\, dx}$$

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f^2(x) - g^2(x))\, dx}{\int_a^b (f(x) - g(x))\, dx}$$

The Slog

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- Vectors

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- Actually I like vectors a lot but I needed something here.

Enlightenment

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- At the end of the quarter we start doing double integrals

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Enlightenment

- Calc 3
- At the end of the quarter we start doing double integrals
- In Calc 2 doing integrals is the armpit of calculus (Lee Singleton).
- In Calc 3 doing double integrals brings Nirvana.

Enlightenment

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$$Avg = \frac{\int\int_R f(x,y)\,dA}{\int\int_R 1\,dA}$$

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NIRVANA!

Enlightenment

Find Average Value of x on region R.

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Enlightenment

Find Average Value of x on region R.

$$\begin{aligned} Avg &= \frac{\int\int_R x\,dA}{\int\int_R 1\,dA} \\ &= \frac{\int_a^b \int_{g(x)}^{f(x)} x\,dy\,dx}{\int_a^b \int_{g(x)}^{f(x)} 1\,dy\,dx} \end{aligned}$$

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Enlightenment

Find Average Value of y on region R.

$$\begin{aligned} Avg &= \frac{\int\int_R y\,dA}{\int\int_R 1\,dA} \\ &= \frac{\int_a^b \int_{g(x)}^{f(x)} y\,dy\,dx}{\int_a^b \int_{g(x)}^{f(x)} 1\,dy\,dx} \\ &= \frac{\int_a^b \frac{f^2(x)}{2} - \frac{g^2(x)}{2})\,dx}{\int_a^b f(x) - g(x)\,dx} \\ &= \bar{y} \end{aligned}$$

Enlightenment

Now look at Papus again.

$$V = 2\pi r A$$

Now look at Papus again.

$$V = 2\pi r A$$

$$r = \frac{\frac{V}{2\pi}}{A}$$

Enlightenment

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- So rotate the region around the y-axis

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- So rotate the region around the y-axis
- Calculate the volume by shells.

Enlightenment

$$r = \frac{\frac{V}{2\pi}}{A}$$

- To calculate $\bar{x}$ we want $r = \bar{x}$.
- So rotate the region around the y-axis
- Calculate the volume by shells.
- Lose the factor of 2π.

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$$r = \frac{\frac{V}{2\pi}}{A}$$

- To calculate $\bar{x}$ we want $r = \bar{x}$.
- So rotate the region around the y-axis
- Calculate the volume by shells.
- Lose the factor of 2π.
- Divide by the area.

Enlightenment

$$r = \frac{\frac{V}{2\pi}}{A}$$

- To calculate $\bar{x}$ we want $r = \bar{x}$.
- So rotate the region around the y-axis
- Calculate the volume by shells.
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$$\bar{x} = \frac{\int_a^b x(f(x) - g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

Enlightenment

$$r = \frac{\frac{V}{2\pi}}{A}$$

- To calculate $\bar{y}$ we want $r = \bar{y}$.
- So rotate the region around the x-axis
- Calculate the volume by washers.
- Lose the factor of 2π.
- Divide by the area.

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f^2(x) - g^2(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

Questions?

Dr. William T. Webber
Whatcom Community College
wwebber@whatcom.ctc.edu