

2004 TOTOM Problems

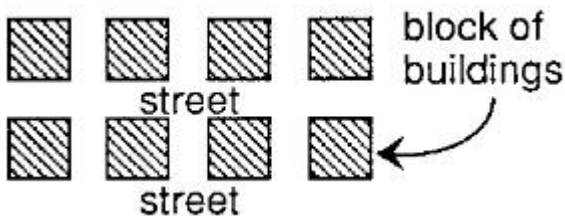
- Submitted by John Engelhardt

A prime number has exactly two distinct factors. A counting number has exactly 3 distinct factors if and only if it is the square of a prime. What counting numbers have exactly 4 distinct factors? 5 distinct factors? N distinct factors?

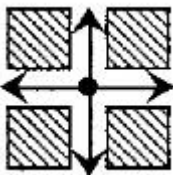
- Some counting numbers can be decomposed into staircases: $15 = 7 + 8$ or $4 + 5 + 6$, or $1 + 2 + 3 + 4 + 5$. Determine which counting numbers can be decomposed into staircases, and in how many ways can it be done for a given number. Explore!

- (From PSU Problem Solving for Middle School Teachers)

Imagine a city whose streets form a square grid, the sides of each being 100 meters long like this.



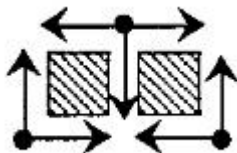
New York City on Manhattan Island is rather like this. Suppose that a policeman is standing at a street corner and that he can spot a suspicious at 100 meters. So he can survey a maximum of 400 meters of street length, like this.



If we have a single block, with 4 corners, we need 2 policemen:



Two blocks in a row will need 3 policemen:



- How many policemen are needed for 3 blocks? 4 blocks? 5 blocks?
- What about blocks in squares? In rectangles? Investigate further and see if you can find some rules.

4. Submitted by Gary Parker

I will flip a coin until I get heads on, say, the N th flip, but I won't tell you what N is. Then I will put 3^N dollars in one box, and 3^{N+1} dollars in another box. I'll mix the up the boxes and open one that you choose so that you can see what is inside. I will give you the money in the open box, or I will give you the money in the other box if you would like to switch. Find an optimal strategy.

5. Submitted by Mike Shaughnessy

(a) Which whole numbers (integers) can be written as a consecutive sum of other whole numbers (integers)?

For example, $15 = 4 + 5 + 6$; $21 = 10 + 11$

(b) Given a whole number (integer), how many different ways are there that it can be written as a consecutive sum?

How do you?

For example, $15 = 4 + 5 + 6$, but $15 = 7 + 8$ also.

6. Submitted by Tom Dick

Suppose (a,b,c) is any primitive Pythagorean triple of natural numbers (i.e., $a^2 + b^2 = c^2$ and a, b, c have greatest common factor 1). Show that:

(a) Either a or b (but not both) must be divisible by 3.

(b) Either a or b (but not both) must be divisible by 4.

(c) Exactly one of a, b, c must be divisible by 5.

7. Submitted by Greg Hill

We know that the infinite sum of all the fractions $1/n$ diverges. What if we only add up those fractions $1/n$ that have a finite decimal expansion?