Chart of the Day

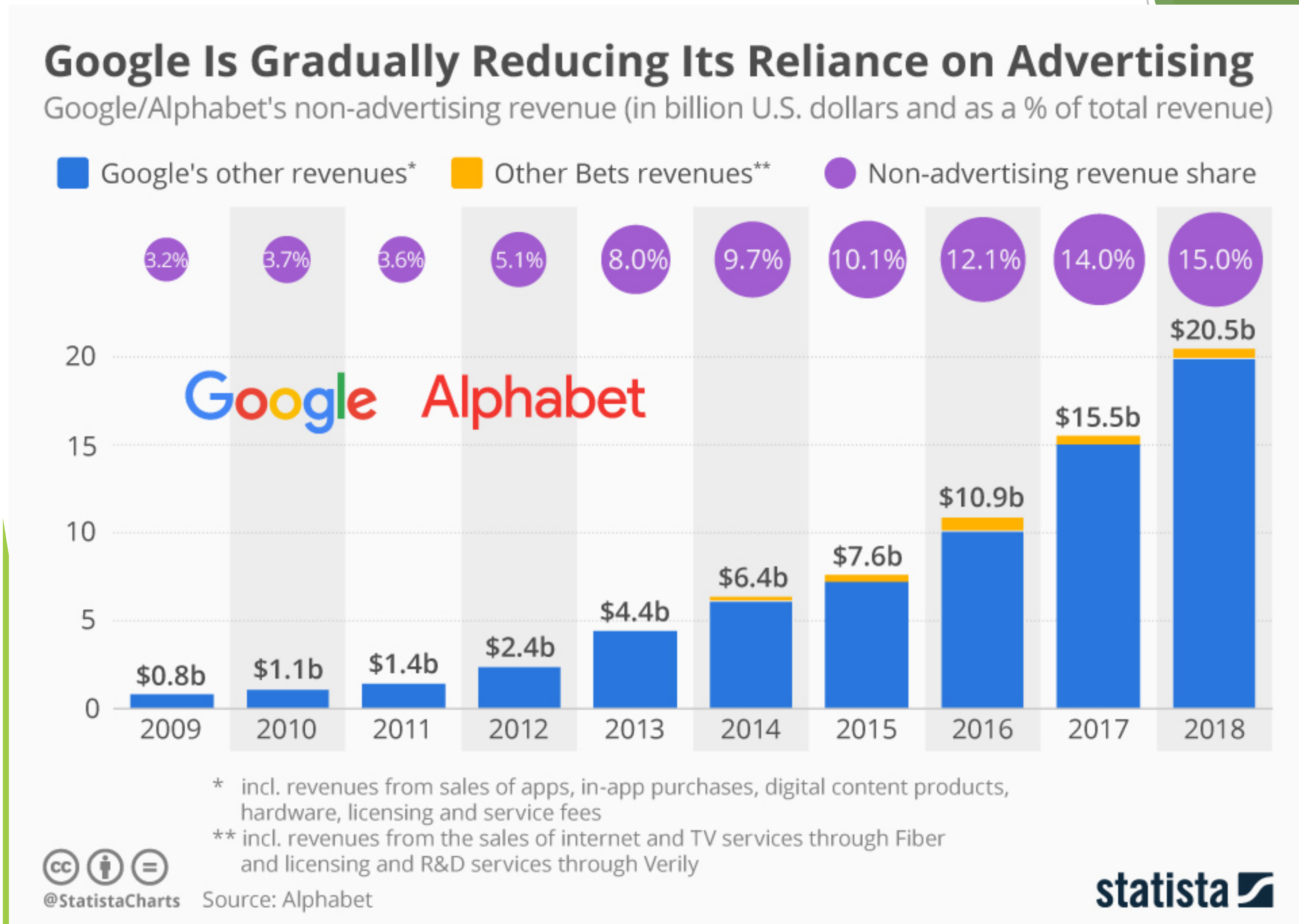
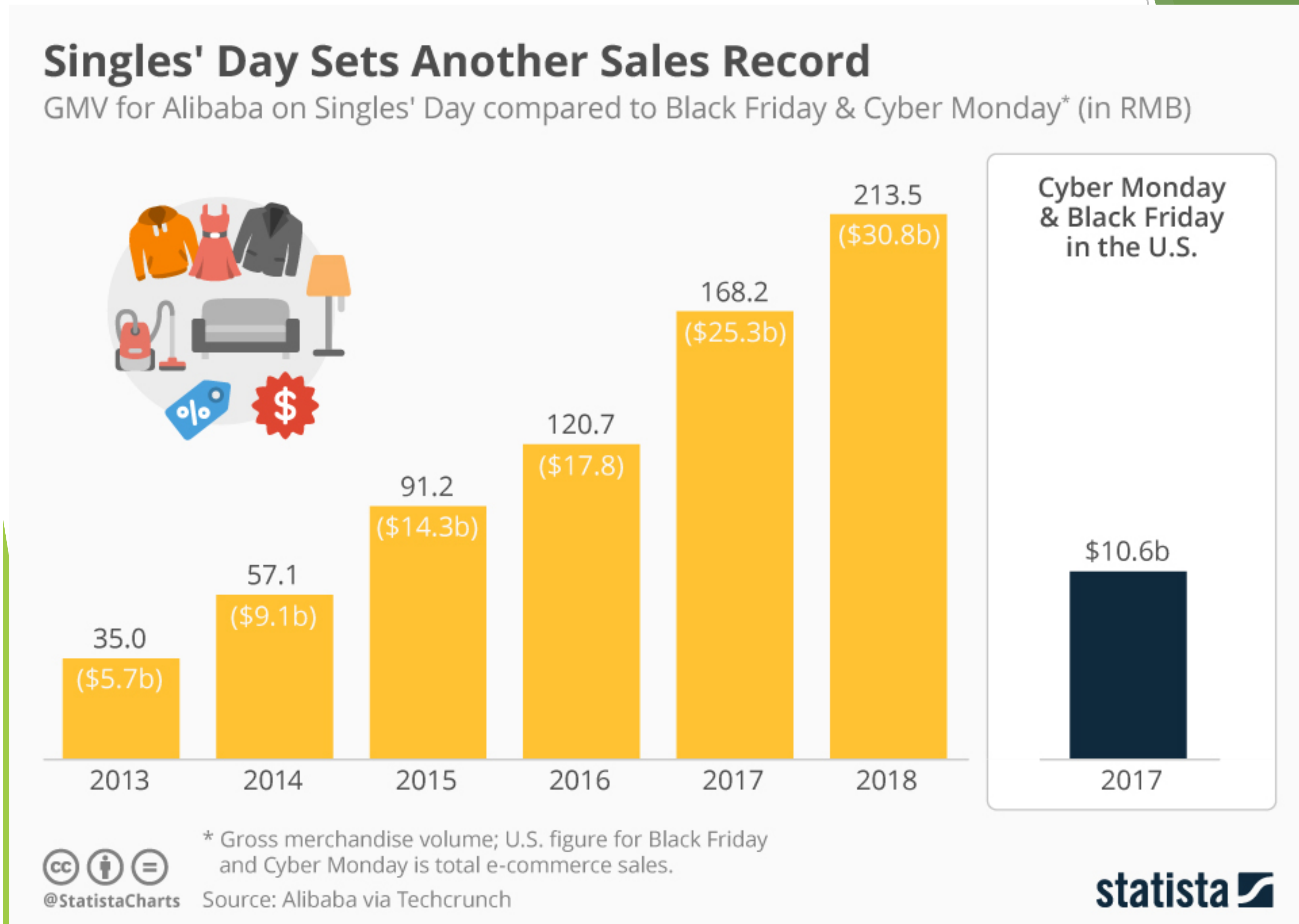


Chart of the Day



Activities to Engage Your Students in the Classroom!

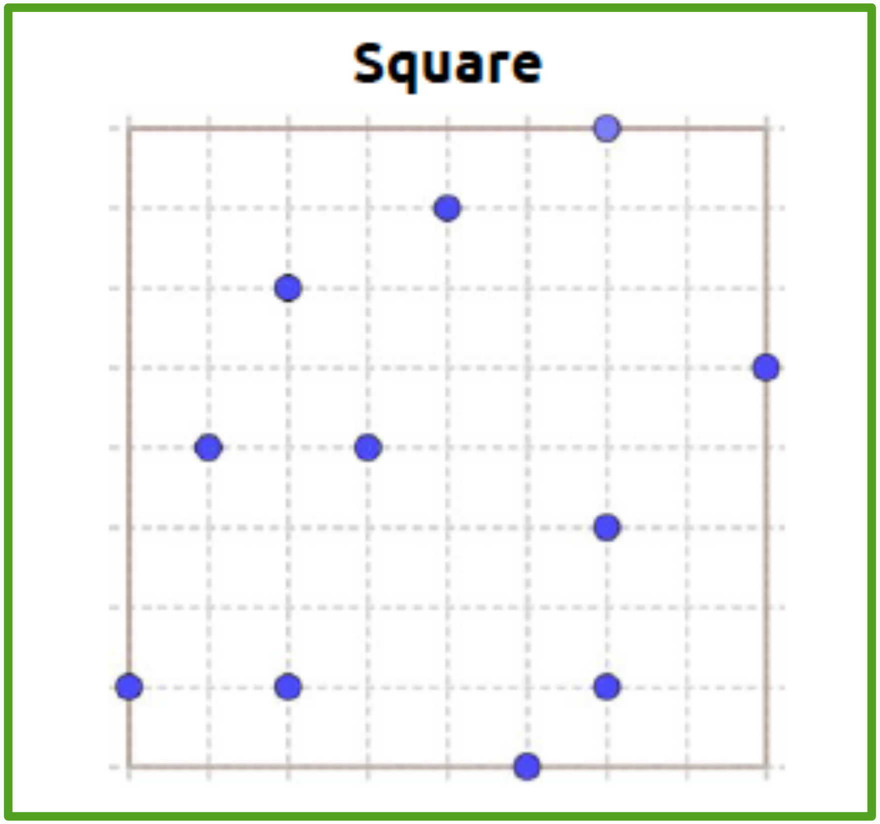
Mark Clark, Palomar College

My definition of a good teacher has since changed from “one who explains things so well that students understand” to “one who gets students to explain things so well that they can be understood.”

Steven C. Reinhart

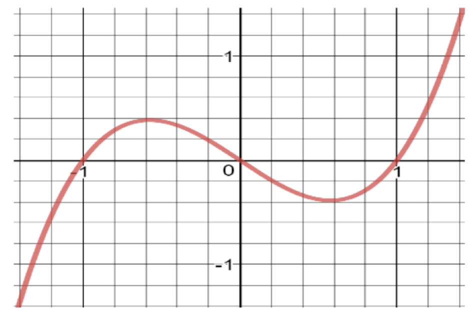
Never Say Anything a Kid Can Say!

Geometry
Zukei Puzzles



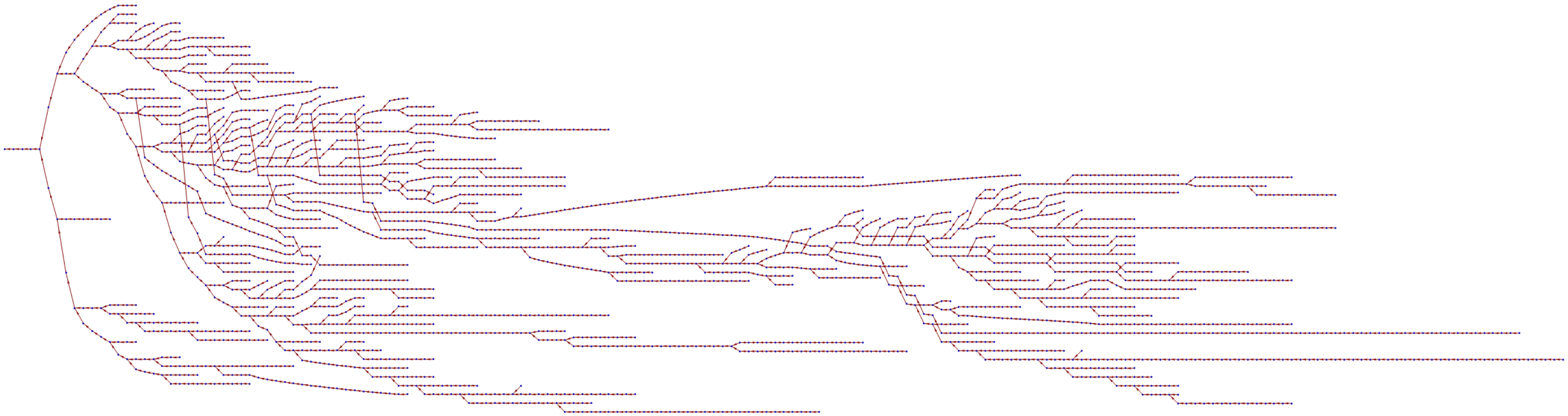
Start with blue page Sukei Puzzles #1
then try green page Sukei Puzzles #3

Polynomial Dominos

$f(x) = \frac{1}{4}(x+2)^2(x-2)^2$	

https://www.glendale.edu/academics/academic-divisions/mathematics/faculty-resources/teaching-and-learning-resources/precalculus-active-learning-labs

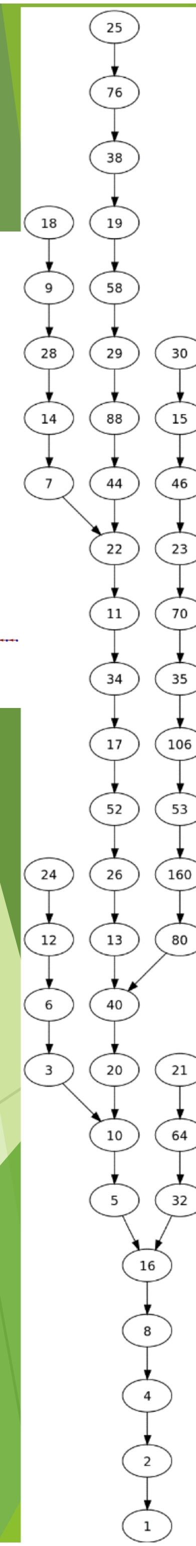
Hailstone Numbers



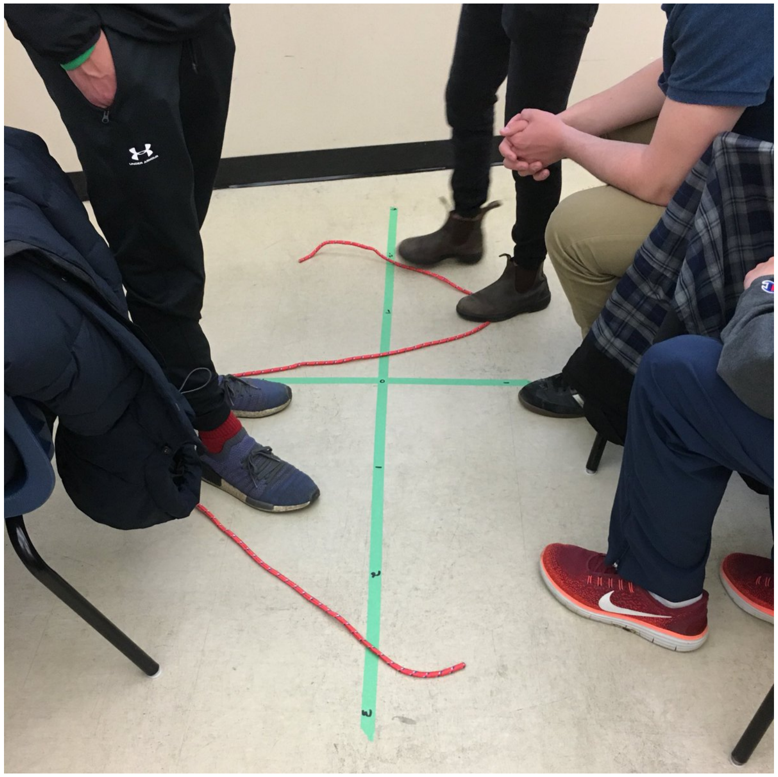
- If the number is even, divide it by two
- If the number is odd, triple it and add one

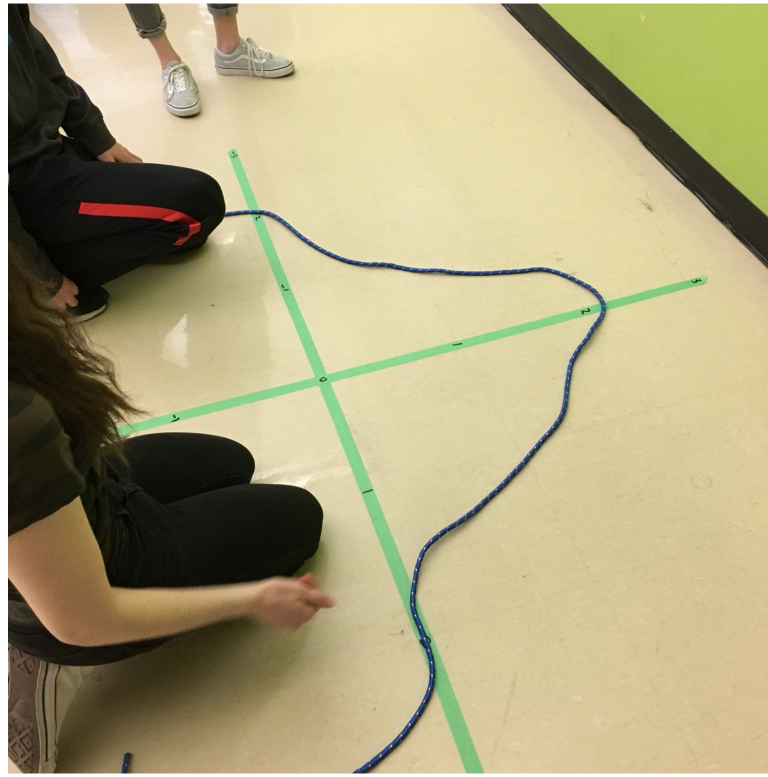
Try it starting with 6

http://en.wikipedia.org/wiki/Collatz_conjecture
https://www.youtube.com/watch?v=5mFpVDpKX70
https://www.jasondavies.com/collatz-graph/



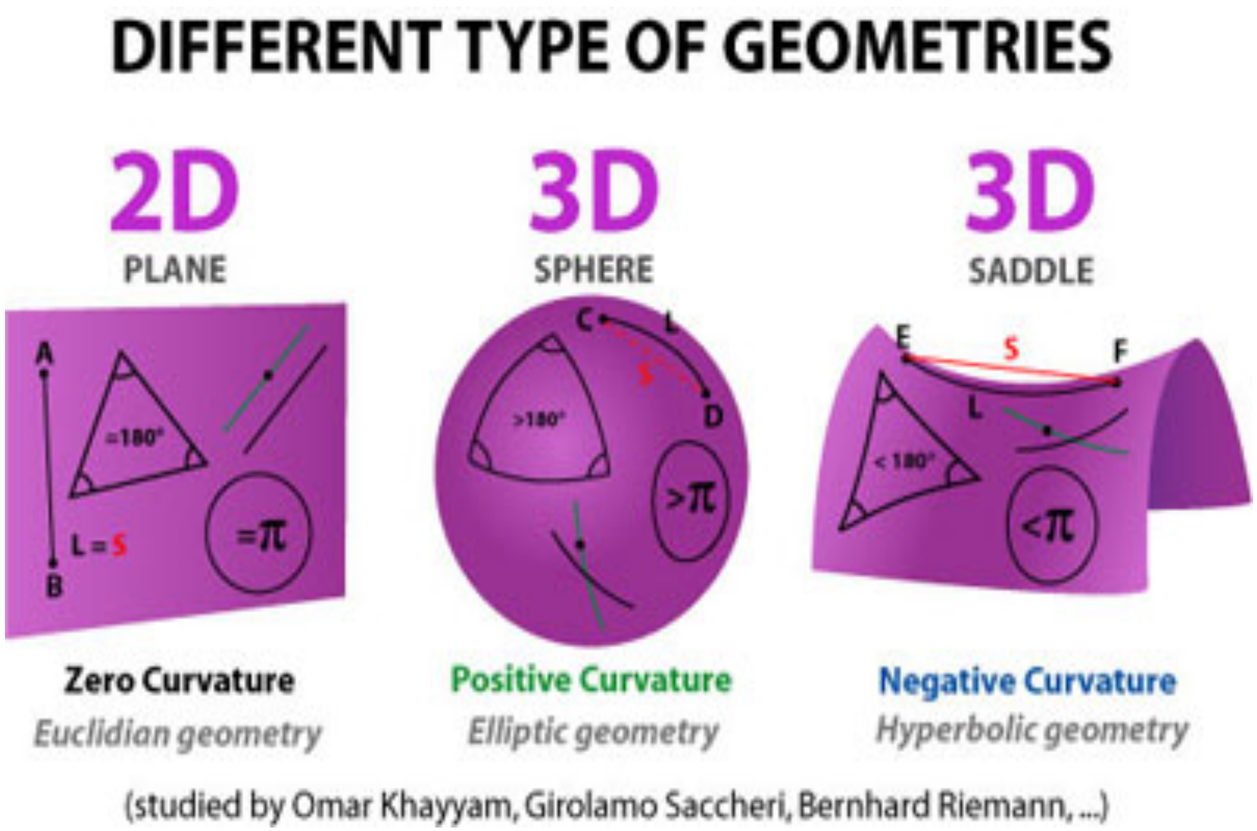
Curve Sketching on the Floor





Sorry I did not bring the ropes!

Non-Euclidean Geometry



Be a really smart ant!

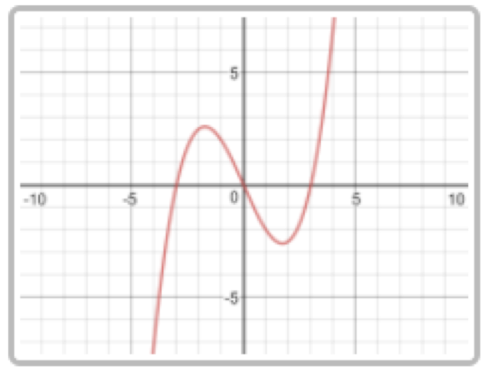
Odd Even or Neither Card Sort

$$f(x) = -x^6 + 5x^2$$

Odd

Even

Neither



x	$f(x)$
-100	56
-10	-23
-1	5
0	0
1	-5
10	23
100	-56

Exponential Function Word Problems Stations Maze

Let's get up and move!

Count off 1 through 10

Go to your station number

Teachers Pay Teachers
Mrs. E Teaches Math!

Natural Logarithms Equations Maze

Can you find your way?

www.teacherspayteachers.com/Store/Jean-Adams
www.flamingomath.com
www.pintrest.com/jeanfaye/

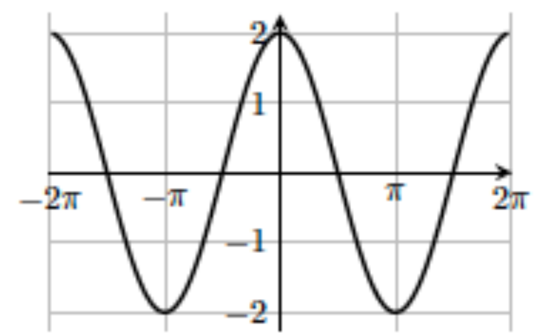
Transformation Matching

$2f(x)$

2

$2\cos(x)$

I

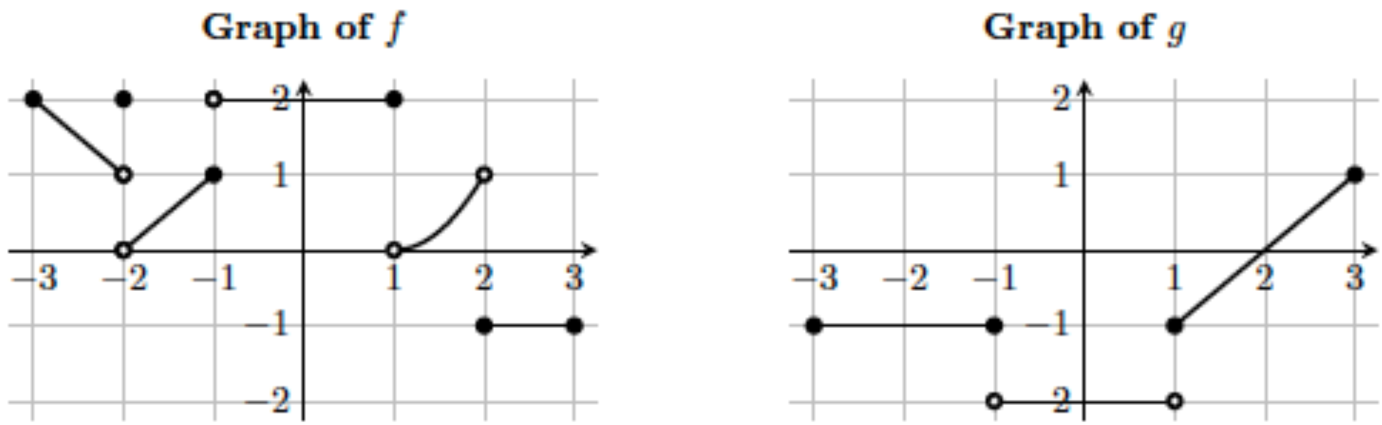


∞

http://math.Colorado.edu/activecalc/

Wacky Limits

These limits are wacky. Help me understand the key. All I have is the answers and not the reasons why the answers are what they are. Do this by providing the correct mathematical reasons/work explaining how one gets the correct answer.



1. $\lim_{x\to 0}(f(x)+g(x)) = 0$

2. $\lim_{x\to 2^-} \frac{g(x)}{f(x)} = \lim_{x\to 2^+} \frac{g(x)}{f(x)} = \lim_{x\to 2} \frac{g(x)}{f(x)} = 0$

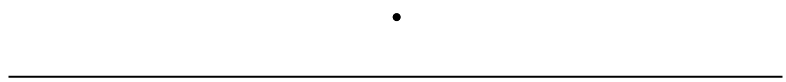
http://math.Colorado.edu/activecalc/

Thank you for joining me today!

Non-Euclidean Geometry

Name________________

Question: Suppose that you have a line on a plane and a point which is not on the line. How many (distinct) lines can you draw that pass through that external point that does not intersect the line?



The essential difference between Euclidean and non-Euclidean geometry is the nature of parallel lines.

For two thousand years, many attempts were made to prove Euclid’s parallel statement (that is that the answer to the previous questions was just one) using Euclid's previous results. The main reason that such a proof was so highly sought after was that, unlike the first results, the parallel statement isn't obvious. If the order the results were listed in his book is significant, it indicates that Euclid included this result only when he realized he could not prove it or proceed without it. It is clear that the parallel statement is different from the other four. It did not satisfy Euclid and he tried to avoid its use as long as possible - in fact the first 28 propositions of The Elements are proved without using it.

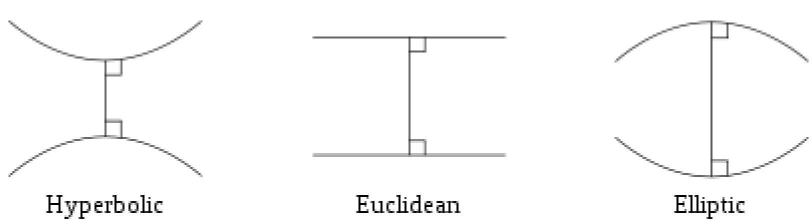
The beginning of the 19th century would finally witness decisive steps in the creation of non-Euclidean geometry. The first works where made by Hungarian mathematician János Bolyai and the Russian mathematician Nikolai Ivanovich Lobachevsky, although Carl Friedrich Gauss claimed that he had developed such geometry about 20 years before, though he did not publish.

Another way to describe the differences between these geometries is to consider two straight lines indefinitely extended in a two-dimensional plane that are both perpendicular to a third line:

- In **Euclidean geometry** the lines remain at a constant distance from each other even if extended to infinity, and are known as parallels.

- In **hyperbolic geometry** they "curve away" from each other, increasing in distance as one moves further from the points of intersection with the common perpendicular.

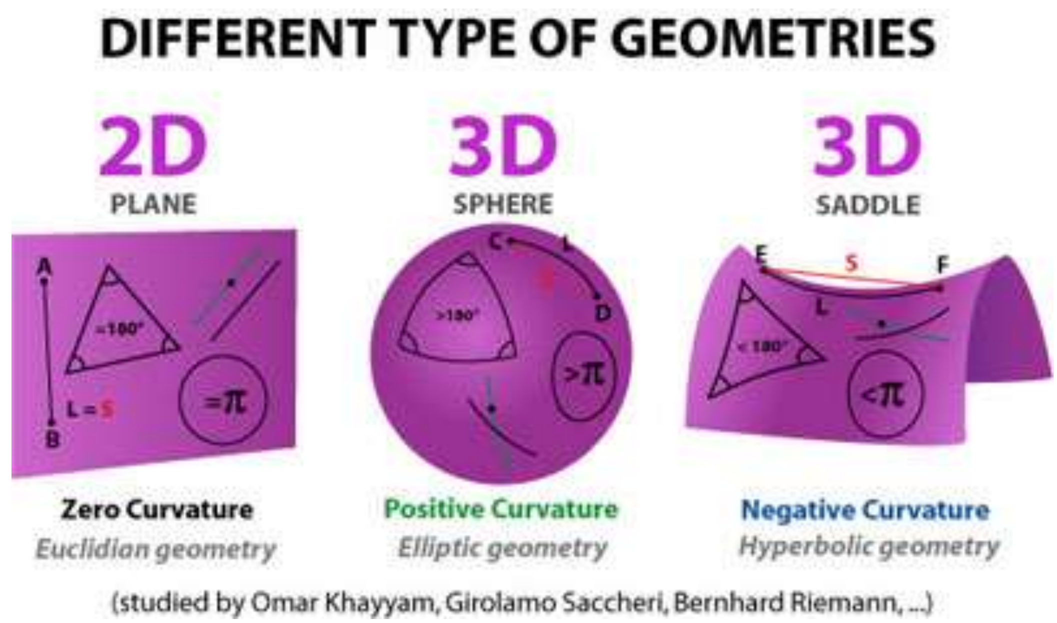
- In **elliptic geometry** the lines "curve toward" each other and eventually intersect.



So the answer to the original question is not straight forwards, as it depends on the geometry you are talking about. In Hyperbolic geometry there are an **infinite** number of distinct lines that pass through the external point and never intersect the line, since they all curve away from the original line. In

Euclidean geometry there is **only one**, the normal one we are accustomed to seeing. Finally in Elliptic geometry there is **none**, because all the lines you draw curve towards the line and thus will eventually intersect it.

Non-euclidean geometry can be understood by picturing the drawing of geometric figures on curved surfaces, for example, the surface of a sphere or the inside surface of a saddle.



Let's compare some differences in these geometries:

	Euclidean	Elliptic	Hyperbolic
Sum of the interior angles of a triangle			
Ratio of a circle's circumference to its diameter.			
Number of distinct lines that meet the parallel statement			
Shape of the shortest path between two points			
Curvature			

http://en.wikipedia.org/wiki/Non-Euclidean_geometry
http://www-history.mcs.st-and.ac.uk/HistTopics/Non-Euclidean_geometry.html

Note: The above geometries can also be defined for higher dimensions, i.e. there is a hyperbolic 3 space etc.
Challenge: Suppose you were an ant of a very smart ant civilization that lived only on the surface of a planet of unknown geometry. The planet has a special force field that prevents you from jumping or elevate away from the ground (no tall buildings, cannot get into rocket and escape force field, etc) and it also prevents us from drilling down any depth. How can you detect what type of geometry the planet has? (For example how can you tell if it's a flat planet vs a sphere planet with a MASSIVELY large radius remembering that you cannot go to "space" and observe the planet from "outside")

Can you describe how your method from the challenge problem be used in our real 3-dimensional world to determine the shape of the universe we live in?

http://en.wikipedia.org/wiki/Shape_of_the_universe
Note: there are three competing possible shapes: Flat, Poincaré dodecahedral space and the Picard horn. In which some models have finite volume but no "edge" if you travel indefinitely in one direction.

Day____ Non-Euclidean Geometry Name___KEY____

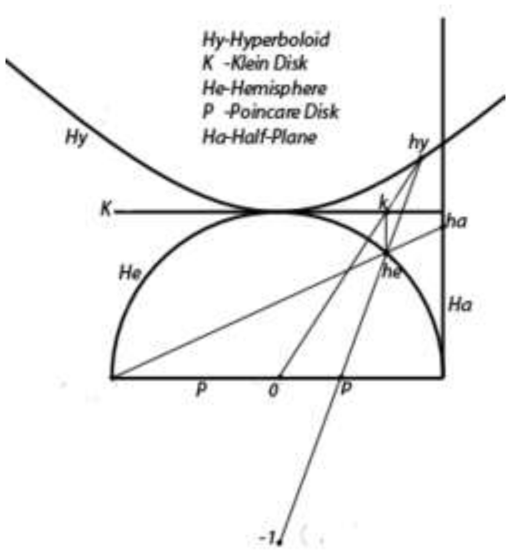
Let's compare some differences in these geometries:

	Euclidean	Elliptic	Hyperbolic
Sum of the interior angles of a triangle	**=180**	**>180**	**<180**
Ratio of a circle's circumference to its diameter.	**$=\pi$**	**$>\pi$**	**$<\pi$**
Number of distinct lines that meet the parallel statement	**One**	**None**	**Infinite**
Shape of the shortest path between two points	**Straight line**	**Circle arc**	**Part of hyperbola**
Curvature	**Zero**	**Positive**	**Negative**

http://en.wikipedia.org/wiki/Non-Euclidean_geometry
http://www-history.mcs.st-and.ac.uk/HistTopics/Non-Euclidean_geometry.html

Models of the hyperbolic plane.
There are different pseudospherical surfaces that have for a large area a constant negative Gaussian curvature, the pseudosphere being the best well known of them.

But it is easier to do hyperbolic geometry on other models: The Beltrami–Klein model, The Poincaré disc model, The Poincaré half-plane model, The hyperboloid model, The hemisphere model, The Gans model or flattened hyperboloid model.



Poincaré disk, hemispherical and hyperboloid models are related by stereographic projection from −1. Beltrami–Klein model is orthographic projection from hemispherical model. Poincaré half-plane model here projected from the hemispherical model by rays from left end of Poincaré disk model.

Question: Why do YOU think it was so hard, and took more than two thousand years for people to "prove" the fifth postulate out of the first four?

Answers can vary, just making sure to stimulate interest in the complexity of the problem, not how smart the people attempting the problem.
Many "failed" proofs where found to be equivalent to the parallel postulate, but this fact was not immediately obvious.
Lots of hard work, perseverance and learning from mistakes is necessary to tackle hard questions.
If you constantly hit a wall following one path, it might not be the best one to take and searching for an alternative can shed new light to old problem or open new areas of math!!!!!!!

Challenge: How can you detect what type of geometry the planet has? (For example how can you tell if it's a flat planet vs a sphere planet with a MASSIVELY large radius remembering that you cannot go to "space" and observe the planet from "outside")

Construct a very large triangle and measure the angles of it, to see in which geometry the planet lives on. Alternatively construct a very large circle and measure the ratio of its circumference to its diameter to see if it deviates from the exact value of pi.

Can you describe how your method from the challenge problem be used in our real 3-dimensional world to determine the shape of the universe we live in?

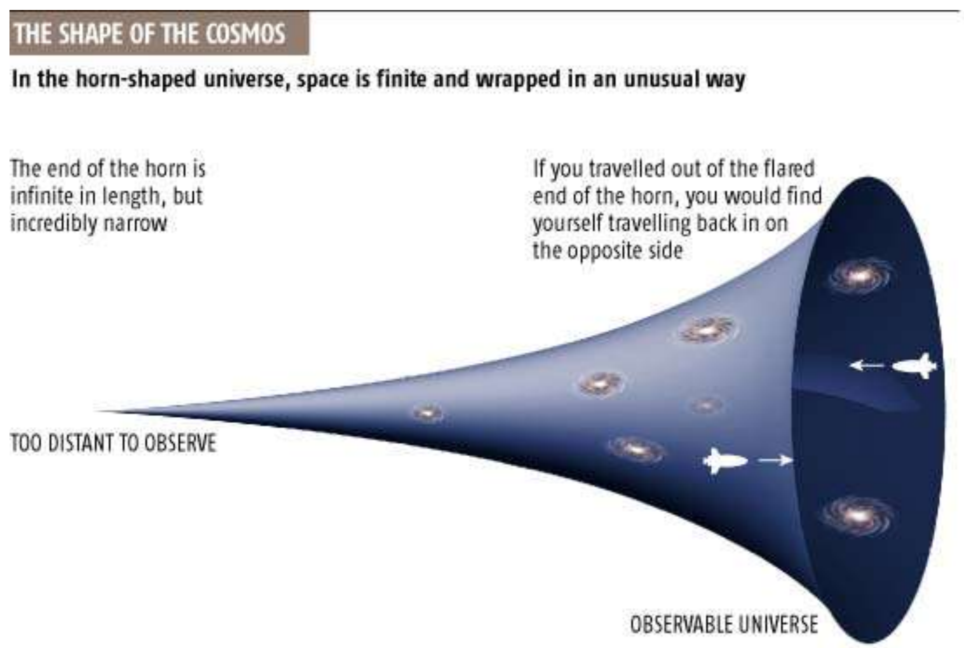
The shape of the global universe can be broken into three categories

1. Finite or infinite
2. Flat (no curvature), open (negative curvature) or closed (positive curvature)
3. Connectivity, how the universe is put together, i.e simply connected space or multiply connected.

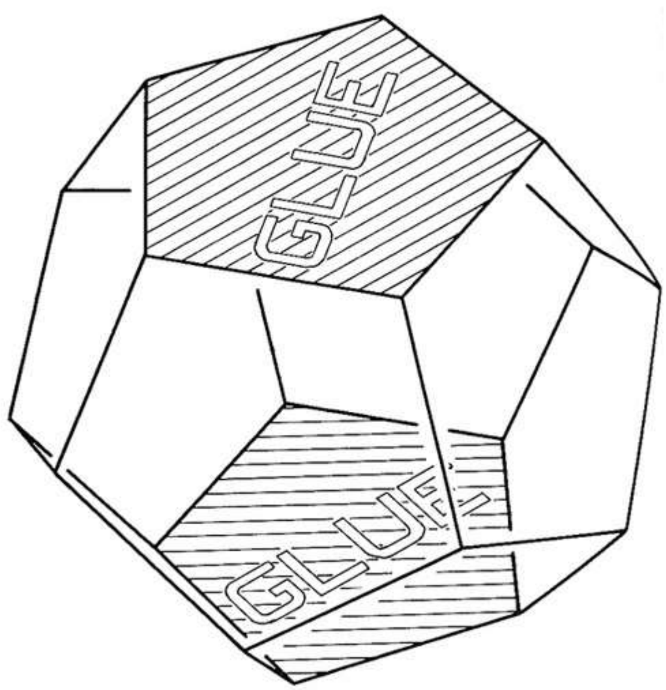
One should note that any combination of these can occur, that is, a flat universe can be finite or infinite, or any combination.

The exact shape is still a matter of debate in matter of debate in physical cosmology, however, experimental data from various, independent sources (WMAP, BOOMERanG and Planck for example) confirm that the universe is flat with only a 0.4% margin of error. Theorists have been trying to construct a formal mathematical model of the shape of the Universe. In formal terms, this is a 3-manifold model corresponding to the spatial section (in comoving coordinates) of the 4-dimensional space-time of the Universe. The model most theorists currently use is the so-called Friedmann–Lemaître–Robertson–Walker (FLRW) model. Arguments have been put forward that the observational data best fit with the conclusion that the shape of the Universe is infinite and flat, but the data are also consistent with other possible shapes, such as the so-called Poincaré dodecahedral space and the Picard horn.

http://en.wikipedia.org/wiki/Shape_of_the_universe



http://www.newscientist.com/article/dn4879-big-bang-glow-hints-at-funnelshaped-universe.html#.VW--O89ViBI

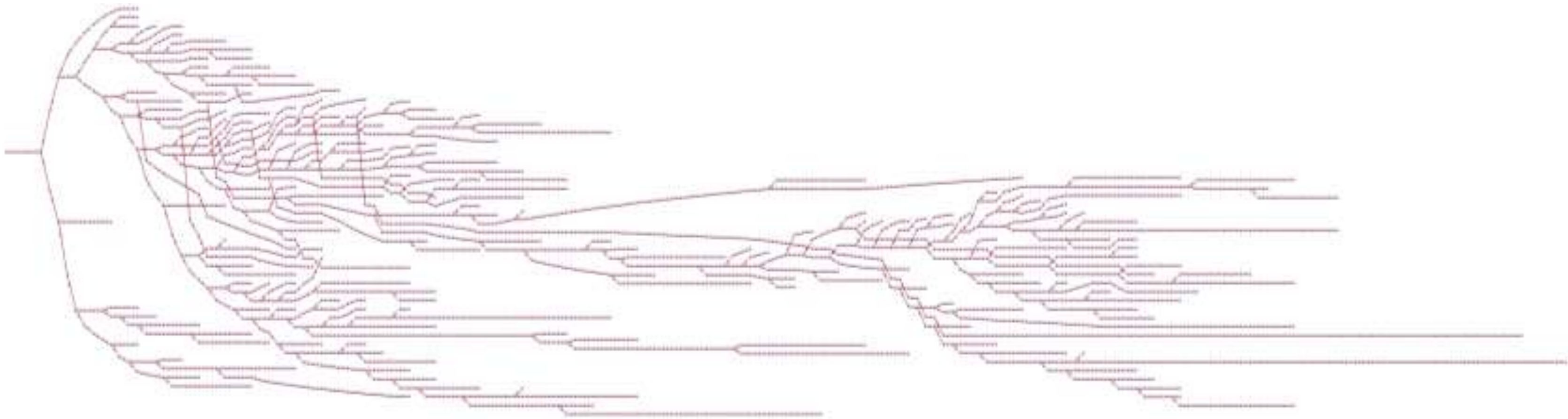


In a Poincare Dodecahedral space every face of a dodecahedron is glued to the opposite face with a one-tenth clockwise turn.

http://www.ams.org/notices/200406/fea-weeks.pdf

Hailstone numbers

Name___________



The **Collatz conjecture** is a conjecture in mathematics named after Lothar Collatz, who first proposed it in 1937. The conjecture is also known as the **3n + 1 conjecture**, or the Syracuse problem; the sequence of numbers involved is referred to as the hailstone sequence or **hailstone numbers**, or as wondrous numbers.

Consider the following operation on an arbitrary positive integer:

- If the number is even, divide it by two.
- If the number is odd, triple it and add one.

Now, form a sequence by performing this operation repeatedly, beginning with any positive integer, and taking the result at each step as the input at the next.

For instance, starting with $n = 6$, one gets the sequence: 6, 3, 10, 5, 16, 8, 4, 2, 1.

http://en.wikipedia.org/wiki/Collatz_conjecture

Find the hailstone sequence for n=11:

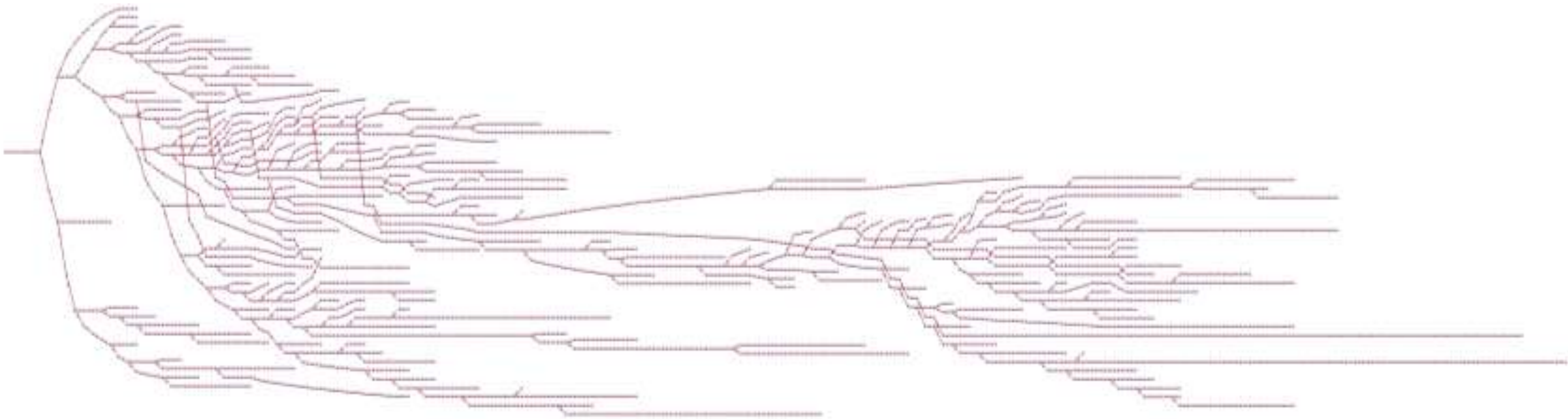
Choose any 3 values (preferably less than 20) and find their hailstone sequence:

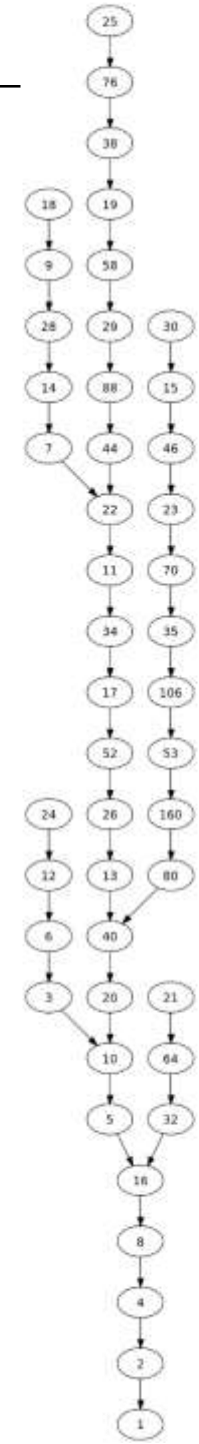
Challenge: Find the hailstone sequence for n=27

Can you see a pattern? Can you guess at what is the conjecture?

Hailstone numbers

Name____Key________





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http://en.wikipedia.org/wiki/Collatz_conjecture https://www.youtube.com/watch?v=5mFpVDpKX70 https://www.jasondavies.com/collatz-graph/

Find the hailstone sequence for n=11:

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Choose any 3 values (preferably less than 26) and find their hailstone sequence:

Answers can vary from students' choices:

7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

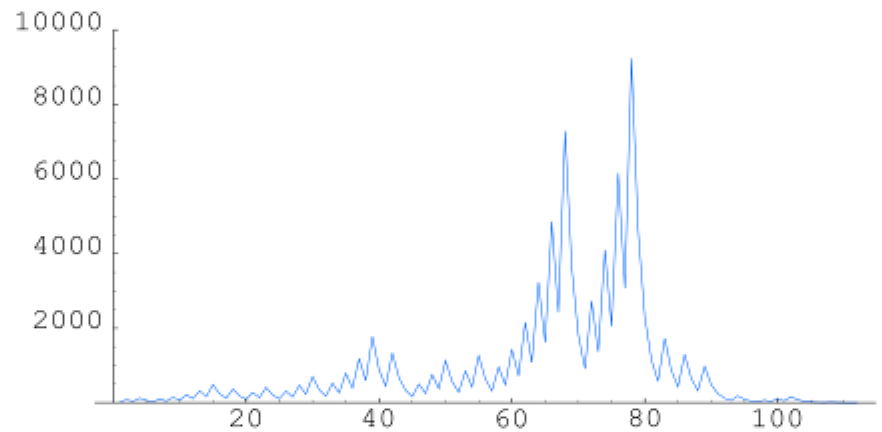
12, 6, 3, 10, 5, 16, 8, 4, 2, 1

15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2,1

19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

21, 64, 32, 16, 8, 4, 2, 1

25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1



Challenge: Find the hailstone sequence for n=27

27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, **9232**, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

The sequence for n = 27, listed and graphed above, takes 111 steps (41 steps through odd numbers), climbing to a high of 9232 before descending to 1

Can you see a pattern? Can you guess at what is the conjecture?

This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

Unsolved problem in mathematics:

Does the Collatz sequence eventually reach 1 for all positive integer initial values?

[(more unsolved problems in mathematics)]

WACKY LIMITS

Purpose. The purpose of this activity is to help students understand deeply what it means for a limit to exist. It also enforces understanding of limit laws, composition of functions, one-sided limits, and the notion of no limit existing.

Preparation (before class) and implementation (in class). This can either be used as a follow-up activity for Limit Sentences or as a stand-alone activity.

Print a copy of the Wacky Limit worksheet for each student. We suggest randomly collecting one per group at the end of the period. Tell the students that this will be the case, so they know to work together yet also document their work individually.

Emphasize the instructions to the group (or have the students read the directions aloud). The goal is to have the students explain why the answers are the way they are using mathematically correct language and notation. Note that many of the answers are not easy. In fact, they may even make you think. We strongly suggest you try the activity before giving it to your students!!!

Suggested directions. If you wish to make this project as discovery-based as possible, you can distribute the activity, or have it waiting on students' tables as they come in, without instructions. Alternatively, you can introduce the project with directions like the following:

Leading questions and general ideas. As the students explore this activity, certain questions, like the following, may arise—or you may wish to bring them up to guide the students in their learning.

-
-
-
-
-

Debrief. If possible, leave some time after the activity is completed for discussion that is more content-focused. This will provide students with the opportunity to understand how the explorations they have just completed apply to the "nuts and bolts" of the topic in question.

Some issues that might be discussed are:

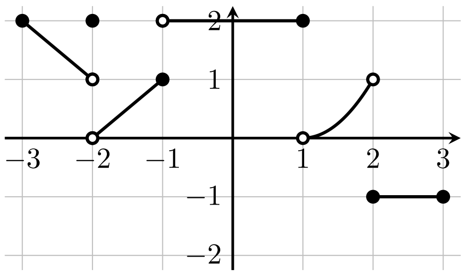
-
-
-
-
-

Follow-up challenge.

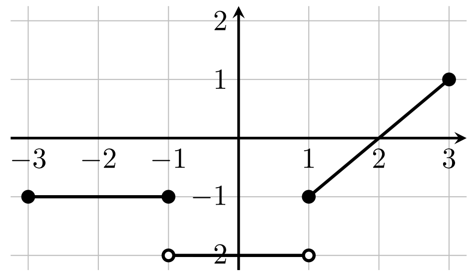
Wacky Limits **Name:** ____________________

These limits are wacky. Help me understand the key. All I have is the answers and not the reasons why the answers are what they are. Do this by providing the correct mathematical reasons/work explaining how one gets the correct answer.

Graph of f



Graph of g



1. $\lim_{x\to 0}(f(x)+g(x)) = 0$

2. $\lim_{x\to 2^-}\frac{g(x)}{f(x)} = \lim_{x\to 2^+}\frac{g(x)}{f(x)} = \lim_{x\to 2}\frac{g(x)}{f(x)} = 0$

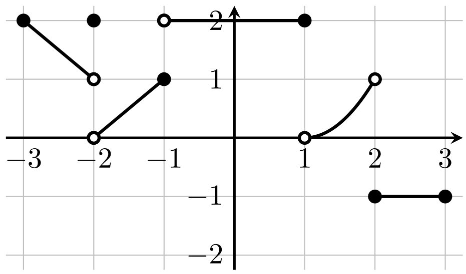
3. $\lim_{x\to -1}(f(x)+g(x)) = 0$

4. $\lim_{x\to -1}\frac{f(x)}{g(x)} = -1$

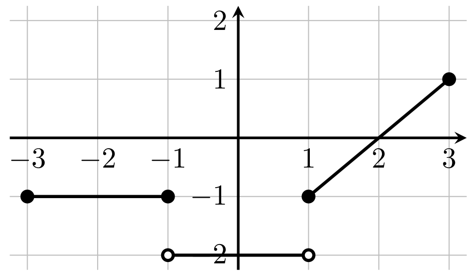
5. $\lim_{x\to 2}(f(x)g(x)) = 0$

6. $\lim_{x\to 3^-} f(g(x)) = 2$

Graph of f



Graph of g



7. $\lim_{x \to 1^+} f(g(x)) = 2$

8. $\lim_{x \to -2^-} g(f(x)) = -1$ (and NOT -2)

9. $\lim_{x \to 1^-} f(g(x)) = 2$ (and NOT 1)

10. $\lim_{x \to 2^-} \frac{f(x)}{g(x)} = -\infty$

11. $\lim_{x \to 2^+} \frac{f(x)}{g(x)} = -\infty$

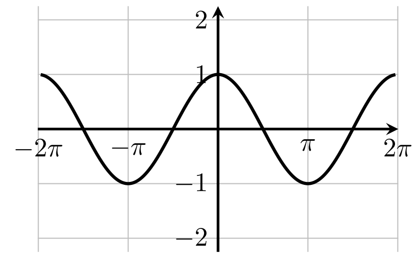
12. $\lim_{x \to 2} \frac{f(x)}{g(x)} = -\infty$

$f(x)$

1

$\cos(x)$

L



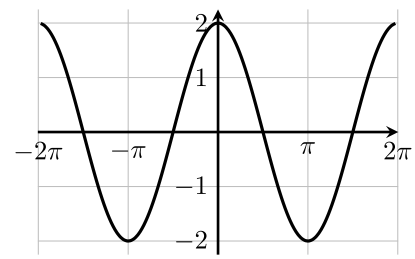
@

$2f(x)$

2

$2\cos(x)$

I



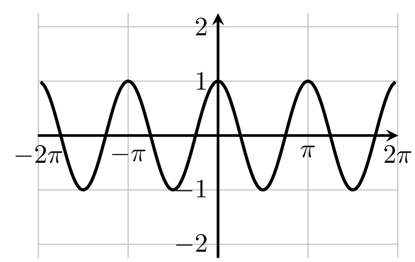
∞

$f(2x)$

3

$\cos(2x)$

A



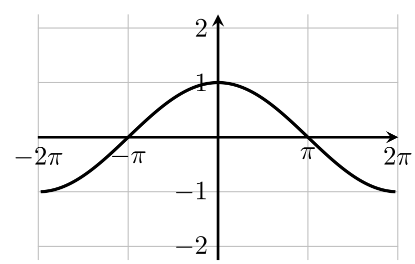
!

$f(\frac{1}{2}x)$

4

$\cos(\frac{1}{2}x)$

F



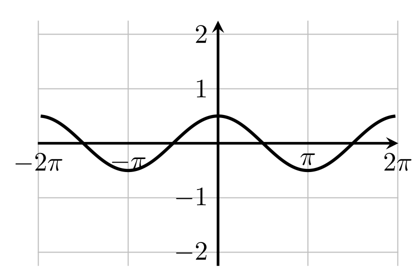
$\triangle$

$\frac{1}{2}f(x)$

5

$\frac{1}{2}\cos(x)$

B



$\square$

Even, Odd or Neither Grouping

Description

Students will sort cards into three groups based on if the card represents a relation that is Even, Odd, or Neither. Students will sort relations that are represented graphically, numerically, and symbolically.

Prep Work

Print out and cut a set of cards for each group of students. Two students per group.

Instructions

○ Pre Activity

Students should have learned how to determine if a relation is even, odd, or neither. This is a review activity. Make sure you have introduced this idea numerically, graphically, and symbolically.

○ Activity

Have students lay out the cards that read Even, Odd and Neither.
Students will then take a card at a time and determine if it is even odd, or neither.
Instructor will walk around to view students work. Ask questions that get students talking about the mathematics. For example, "Why did you put that card in the Even Pile?"
As student finish, have groups check each other's work.

○ Post Activity

Class discussion about the key ideas or tricky things students needed to pay attention to while determining if the cards were even, odd or neither.

Time Required

30 minutes

Alignment

Math 110 – Chapter 5 of OpenStax

x	$f(x)$
-100	56
-10	-23
-1	5
0	0
1	-5
10	23
100	-56

x	$f(x)$
-5	-4
-3	-2
-1	1
1	1
2	-2
3	4

x	$f(x)$
-3	21
-2	-12
-1	-25
0	1
1	-25
2	-12
3	21

x	$f(x)$
-3	-14
-2	8
-1	-3
0	0
1	-3
2	-8
3	14

x	$f(x)$
-3	10.5
-2	2
-1	-0.5
0	-2
1	-0.5
2	2
3	10.5

x	$f(x)$
-5	13
-3	1
-2	-5
0	0
2	5
3	-1
5	-13

Even

Neither

Odd

$f(x) = -x^6 + 5x^2$

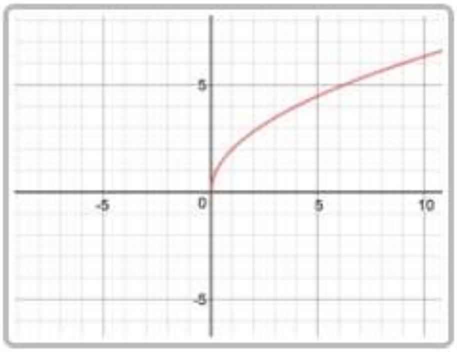
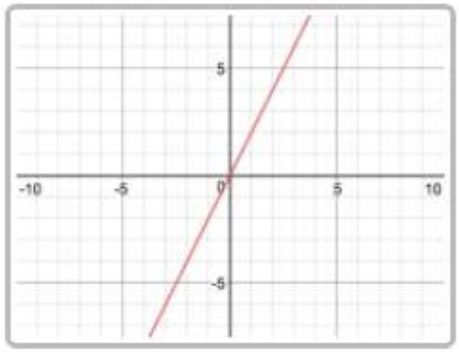
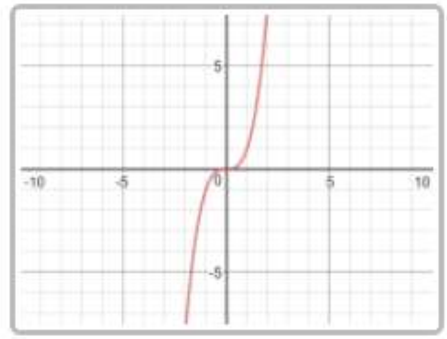
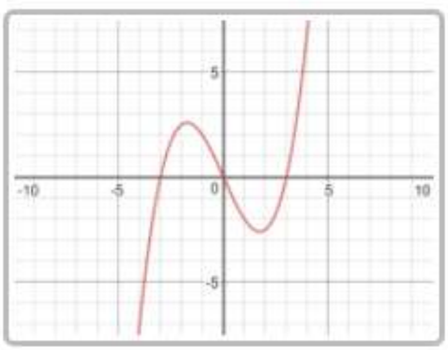
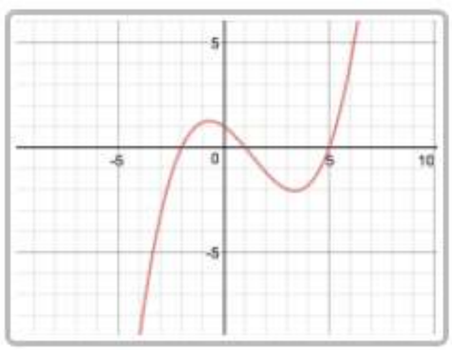
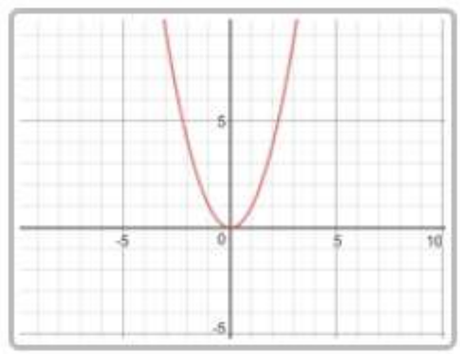
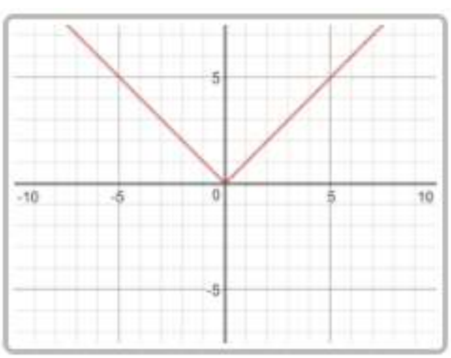
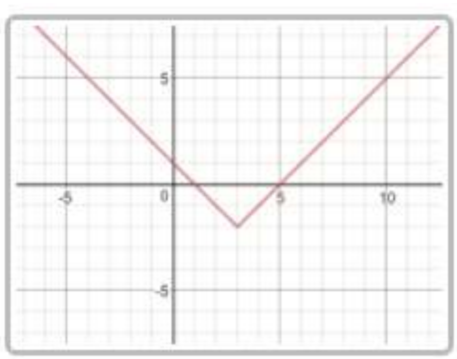
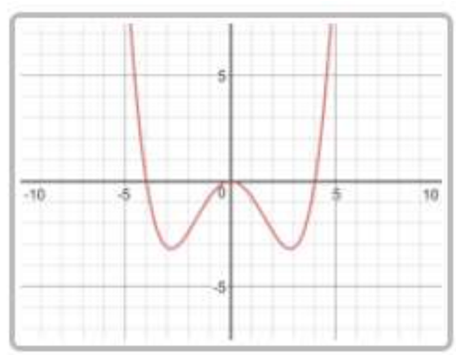
$f(x) = \sqrt{1 - x^2}$

$f(x) = x + 3$

$f(x) = 8 - 2x^2$

$f(x) = 5x$

$f(x) = |x + 2|$

$f(x) = x^3 - 2x$	$f(x) = x^3 - x^2$	$f(x) = \dfrac{1}{x}$
		
		
		

Polynomial Dominos

Description

In this activity, each card has an equation and graph. The students' goal is to match each equation to the correct graph. This is a self-correcting activity that students can start anywhere. If students do not make a mistake, the cards will make a chain where the equation from the first card will match the graph from the last card.

Prep Work

Print out and cut a set of cards for each group of students. Two students per group.

Instructions

○ Pre Activity

Students should have learned the following learning objectives

- End behavior of polynomials
- Multiplicity of zeros of a polynomial
- Number of turning points of a polynomial is the degree of the polynomial minus one.

○ Activity

- Randomly Organize students into groups of two.
- Give each group a set of cards.
- Have students pick on card to start with.
- Have students look for the matching equation to the graph they are looking at.
- Walk around to assist students while they are matching cards. Ask students questions about how they matched cards (correct and incorrect)
- As students finish the activity, have pairs split up to assist groups that are struggling with the activity. Students become the teachers the way.

○ Post Activity

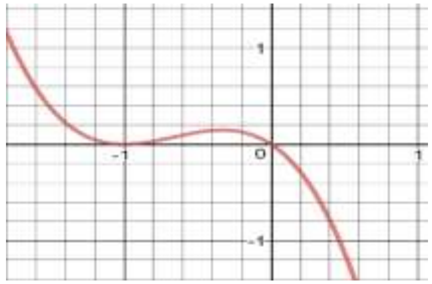
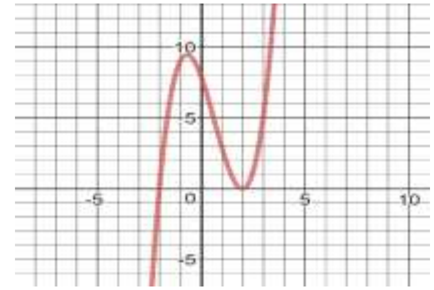
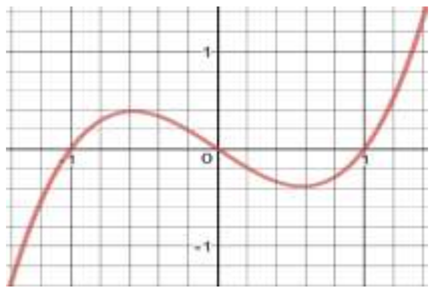
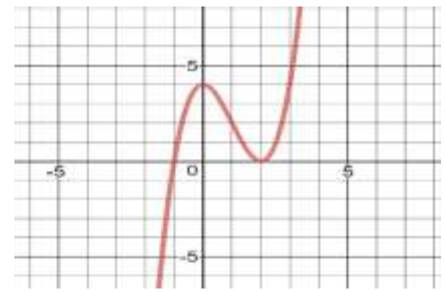
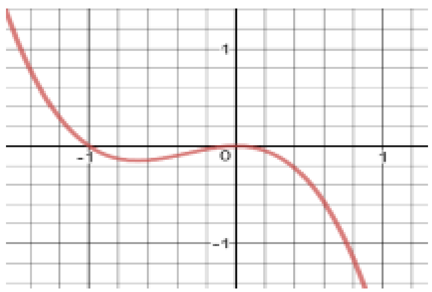
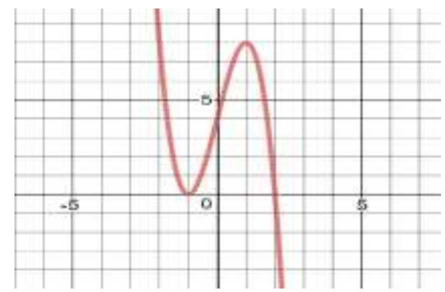
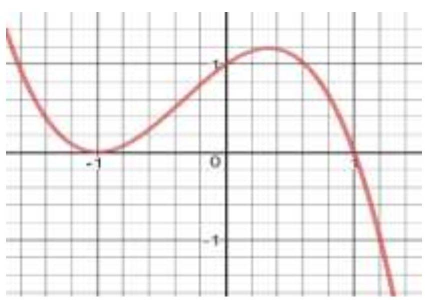
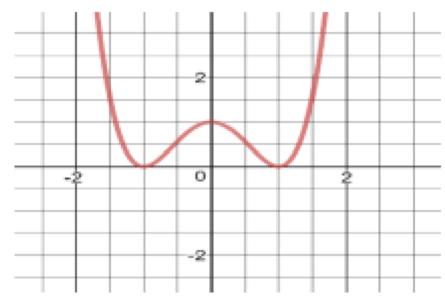
None

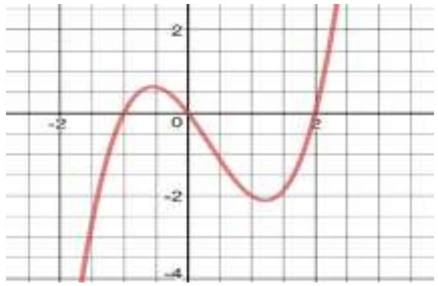
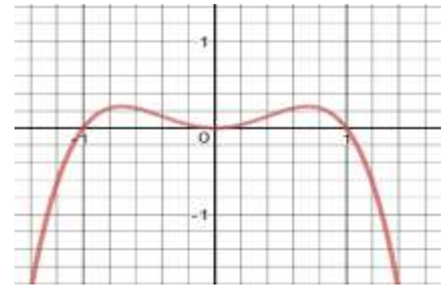
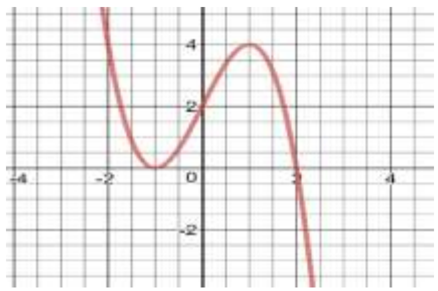
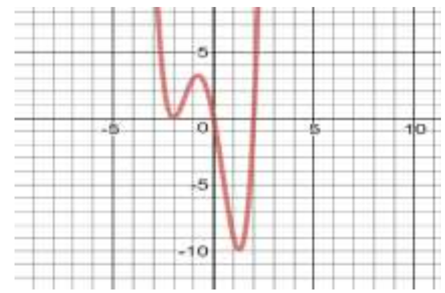
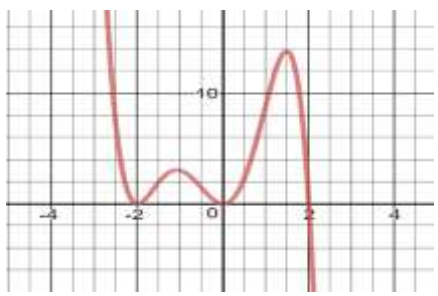
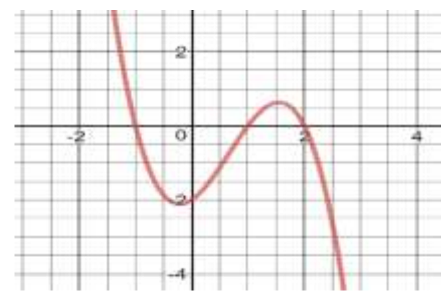
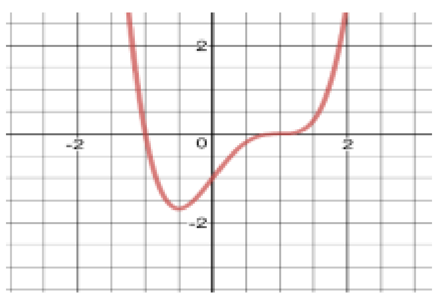
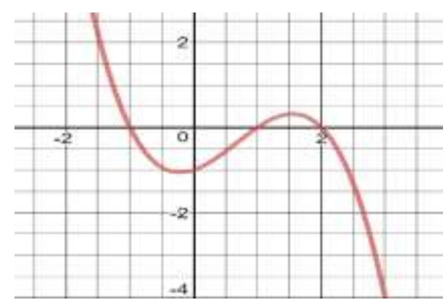
Time Required

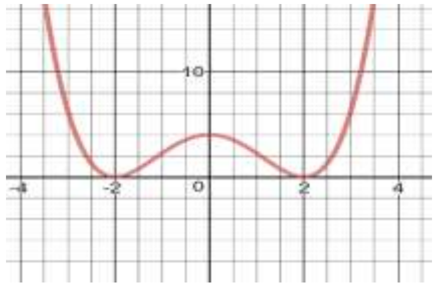
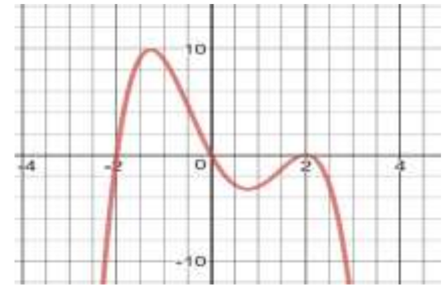
20 minutes

Alignment

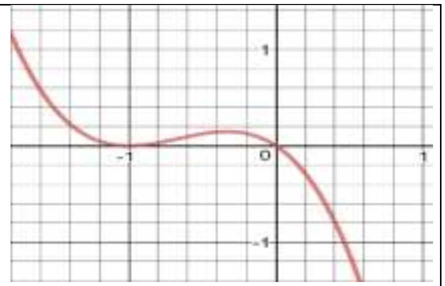
Math 110 - Chapter 5 of College Algebra by OpenStax

$f(x) = -x(x+2)(x-2)^2$		$f(x) = x(x+1)(x-2)$	
$f(x) = \frac{1}{4}(x+2)^2(x-2)^2$		$f(x) = -\frac{1}{2}(x+1)(x-1)(x-2)$	
$f(x) = -x^2(x+2)^2(x-2)$		$f(x) = -x^3 + 2x^2 + x - 2$	
$f(x) = -(x+1)^2(x-2)$		$f(x) = x^4 + 2x^3 - 4x^2 - 8x$	

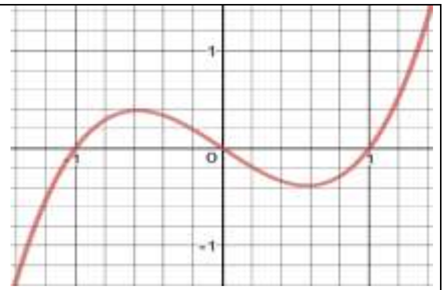
$f(x) = -x^3 - x^2 + x + 1$		$f(x) = x^4 - 2x^2 + 1$	
$f(x) = -x^3 - x^2$		$f(x) = -2x^3 + 6x + 4$	
$f(x) = x^3 - x$		$f(x) = x^3 - 3x^2 + 4$	
$f(x) = -x^4 + x^2$		$f(x) = x^3 - 2x^2 - 4x + 8$	

$f(x) = -x^3 - 2x^2 - x$		$f(x) = x^4 - 2x^3 + 2x - 1$	

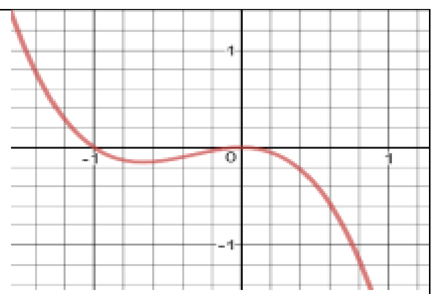
1.

Starting Line	

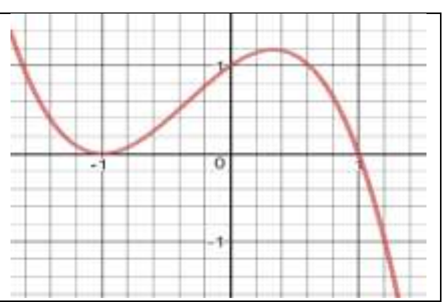
2.

$f(x) = \frac{1}{4}(x+2)^2(x-2)^2$	

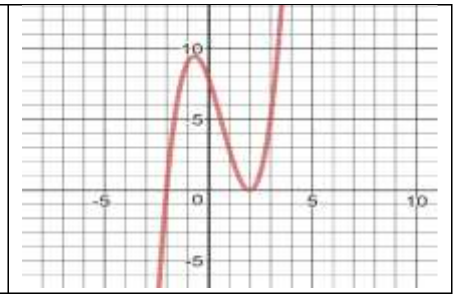
3.

$f(x) = -x^2(x+2)^2(x-2)$	

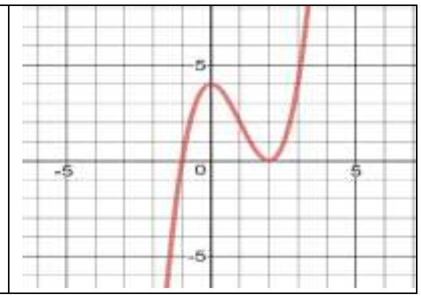
4.

$f(x) = -(x+1)^2(x-2)$	

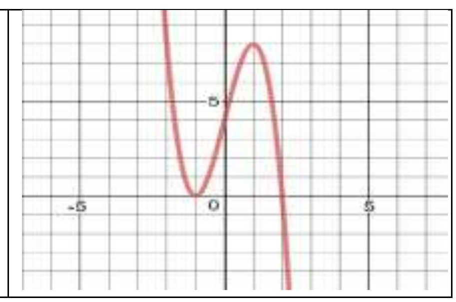
5.

$f(x) = x(x+1)(x-2)$	

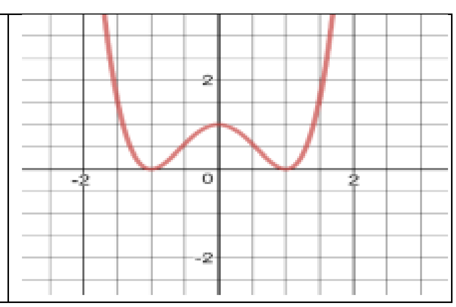
6.

$f(x) = -\frac{1}{2}(x+1)(x-1)(x-2)$	

7.

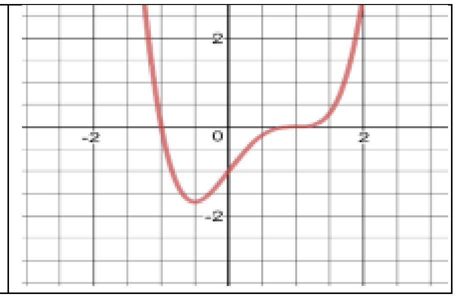
$f(x) = -x^3 + 2x^2 + x - 2$	

8.

$f(x) = x^4 + 2x^3 - 4x^2 - 8x$	

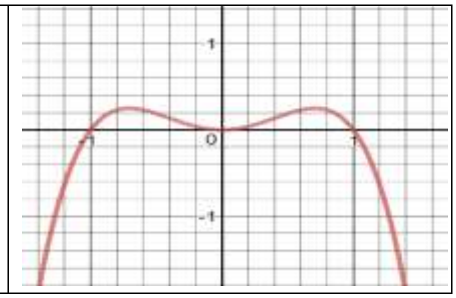
9.

$$f(x) = -x^4 + x^2$$



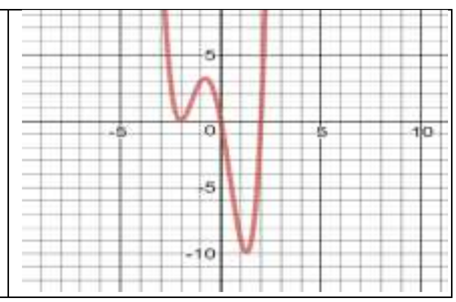
10.

$$f(x) = x^4 - 2x^2 + 1$$



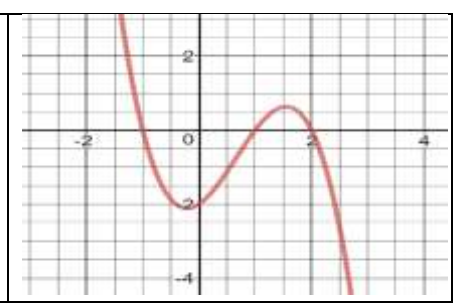
11.

$$f(x) = -2x^3 + 6x + 4$$



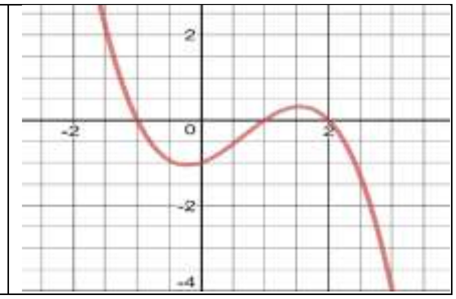
12.

$$f(x) = x^3 - 3x^2 + 4$$



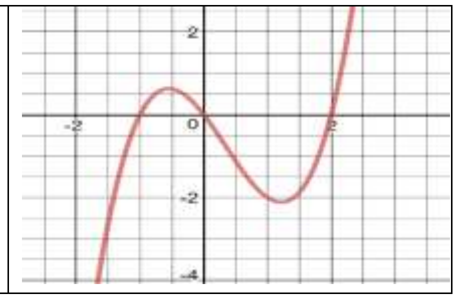
13.

$$f(x) = x^3 - 2x^2 - 4x + 8$$



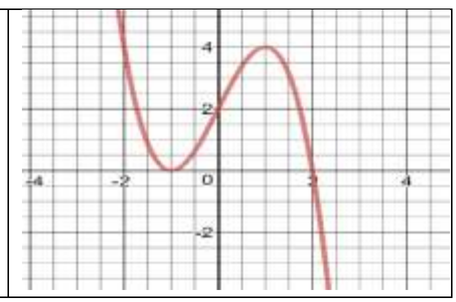
14.

$$f(x) = -x^3 - x^2 + x + 1$$



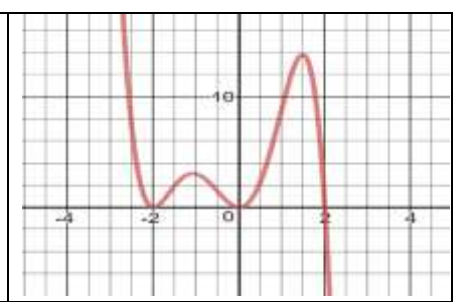
15.

$$f(x) = -x^3 - x^2$$



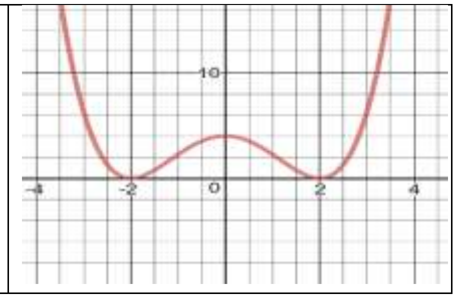
16.

$$f(x) = x^3 - x$$



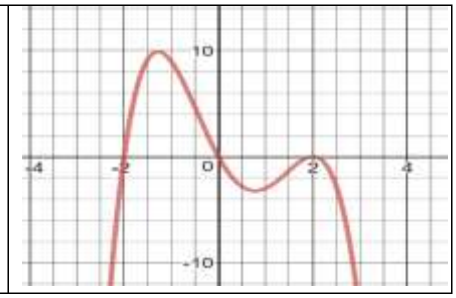
17.

$f(x) = -x^3 - 2x^2 - x$



18.

$f(x) = x^4 - 2x^3 + 2x - 1$



$f(x) = -x(x+2)(x-2)^2$

Finish Line

TRANSFORMATION MATCHING

Purpose. This is a set of three small group activities, intended to get students thinking about, and discussing, function transformations (shifts, dilations, and reflections), from geometric and algebraic perspectives.

One activity entails transformations of a parabola, another transformations of a trig function, and the third transformations of a logarithmic function.

Preparation (before class) and implementation (in class). There is one activity per page. If two or more of these activities are to be used in the same class on the same day, then each page should be printed on paper of a different color, to help keep the separate activities from getting mixed up.

If possible, pages should be laminated, to protect them so that they may be reused.

Each page (activity) is to be cut along the indicated lines, into a set of fifteen cards. In class, each small group should receive a shuffled set of cards.

Suggested directions. If you wish to make this project as discovery-based as possible, you can distribute the cards (or have them already on the students' tables as they come in) without instructions. Alternatively, you can introduce the project with directions like the following:

> "At your table, you will find a set of fifteen cards. The object of this activity is to group the fifteen cards into five groups of three. Each group of three is to contain a "transformation" card (for example, a card that reads $f(x+1)$), a "formula" card (for example, a card that reads $\cos(2x)$), and a "graph" card (for example, a card that depicts the graph of $y = \ln(x)$). That is: match each of your five graph cards with the formula and the transformation that go with it."

If only a portion of a class period is to be used for group work on this topic, then each group can be given just one activity (one sheet; one type of function) to investigate. If more time is available, have different groups start with different activities, and trade activities with other groups as they complete them.

Leading questions and general ideas. As the students explore this activity, certain questions, like the following, may arise—or you may wish to bring them up to guide the students in their learning.

- What strategies did you use to facilitate the sorting? (For example: students might first notice that there are two graphs that are reflections of each other about the x axis; if they know that this corresponds to replacing x with $-x$, they can fairly easily match two triples, making the rest of the matching easier.)
- Is the "parent" function (the function that is simply denoted $f(x)$, or $g(x)$, or $h(x)$) always the most "basic" function? For example: for the parabolas activity, the parent function is $f(x) = (x-1)^2 = x^2 - 2x + 1$, not x^2. Is $f(x) = x^2$ really any more "basic" than $f(x) = (x-1)^2$?
- (If each group has attempted all three activities.) Rank the activities according to degree of difficulty. What makes a given activity harder, or easier, than the others?
- What are some of the algebra techniques and facts that these activities entail? How is the algebra reflected in the graphs?
- (For the logarithm cards.) Why do all five graphs look the same? What's up with that?

Debrief. If possible, leave some time after the activities are completed for discussion that is more content-focused. This will provide students with the opportunity to understand how the explorations they have just completed apply to the "nuts and bolts" of the topic in question.

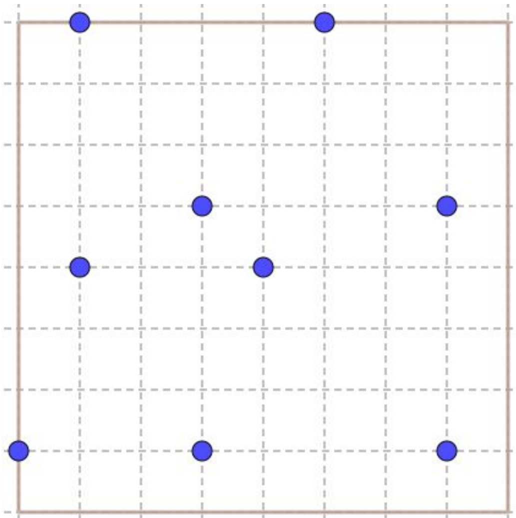
Some issues that might be discussed are:

- Given $y = f(x)$, what is the effect, geometrically speaking, of replacing x by $-x$?
- Given $y = f(x)$, what is the effect, geometrically speaking, of replacing x by ax, where a is a positive constant? A negative constant?
- What about replacing x by $x - k$, where k is constant? How does the sign of k figure in?
- Review basic properties of the logarithm function.
- What questions do you have after completing this activity?

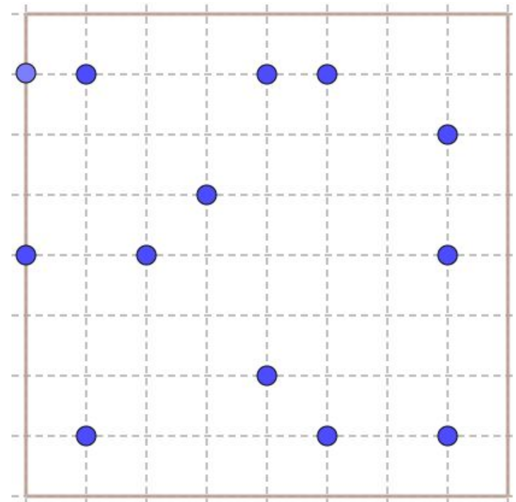
As you discuss these questions, you might want to summarize certain facts on the board. For example: make a list of basic properties of the logarithm function. Construct a table of transformations, written algebraically, versus their geometric effects. And so on.

Zukei Puzzles #1

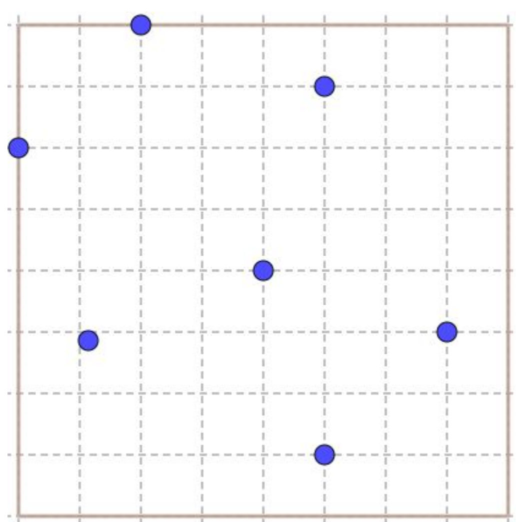
Square



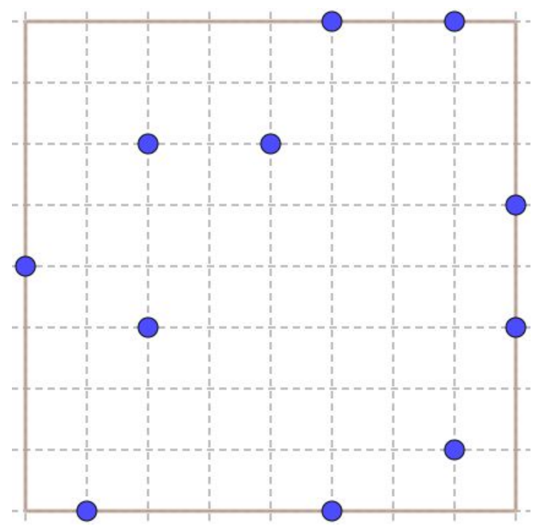
Rectangle



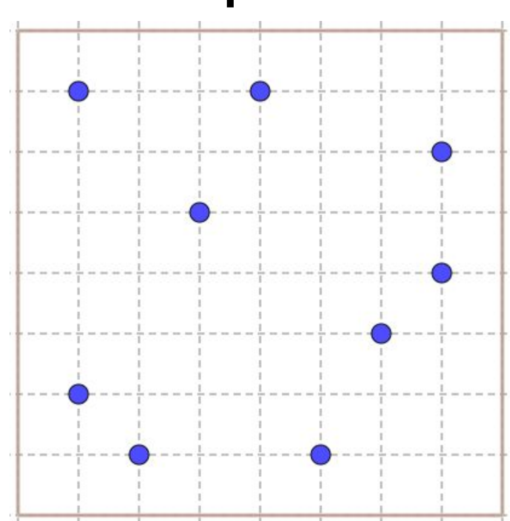
Parallelogram



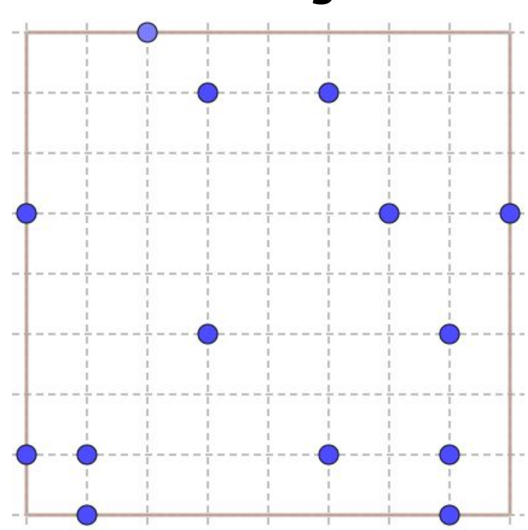
Acute Isosceles Triangle



Trapezoid



Rectangle



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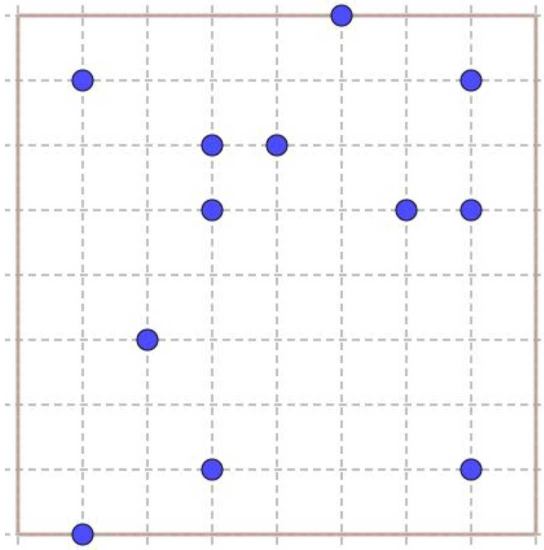
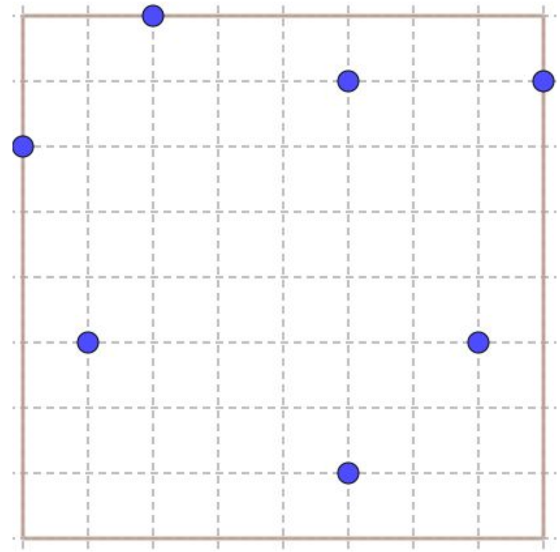
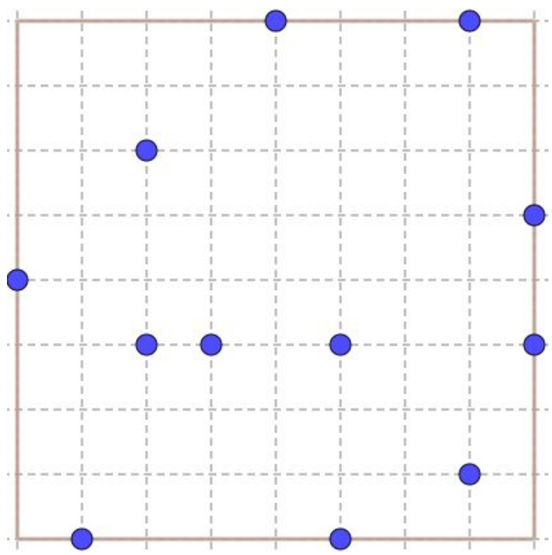
Square



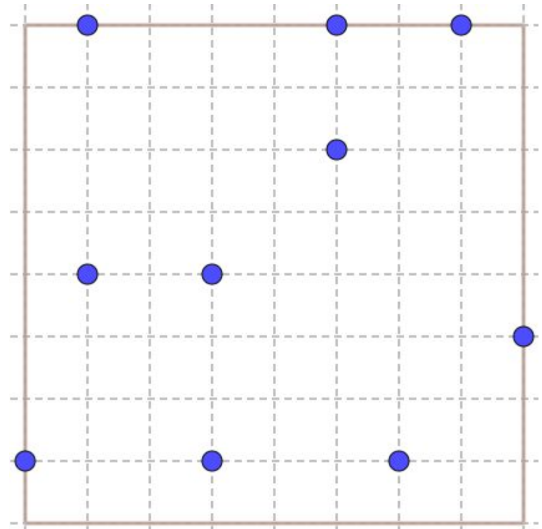
Right Triangle



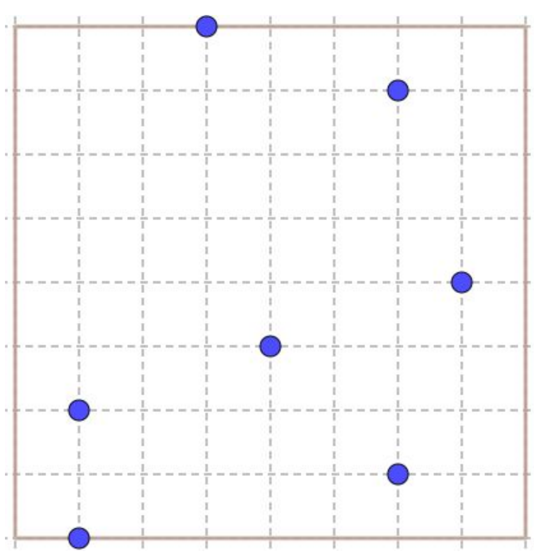
Right Isosceles Triangle



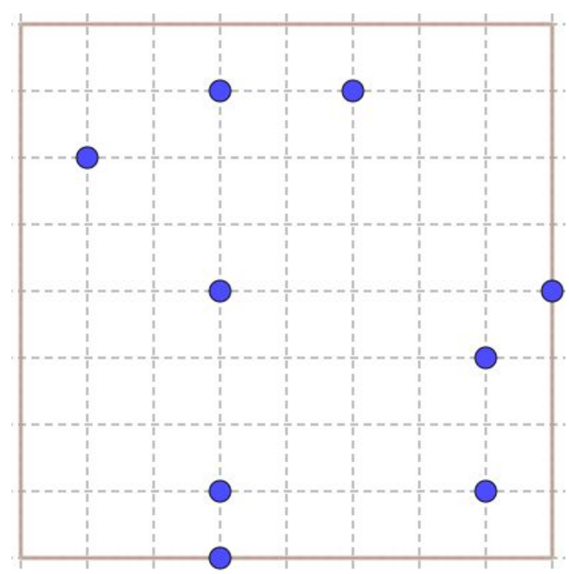
Rectangle



Kite

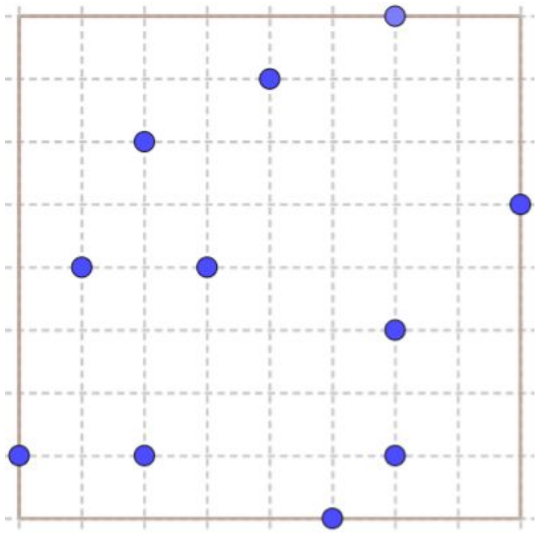


Isosceles Trapezoid

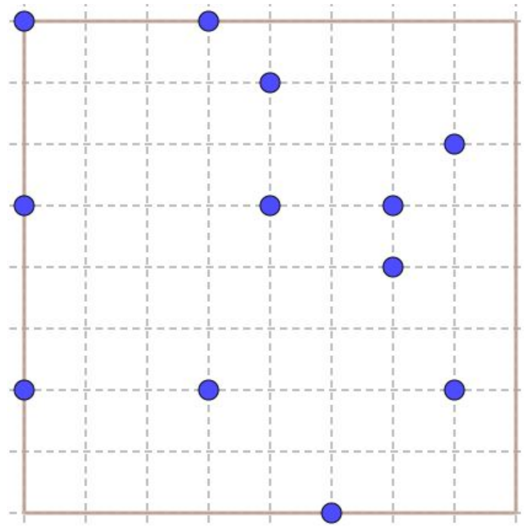


Zukei Puzzles #2

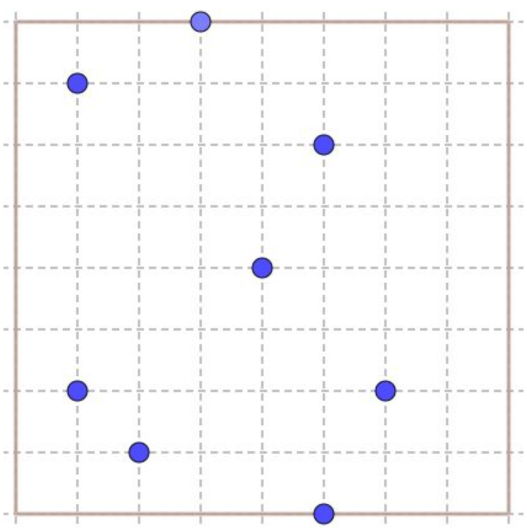
Square



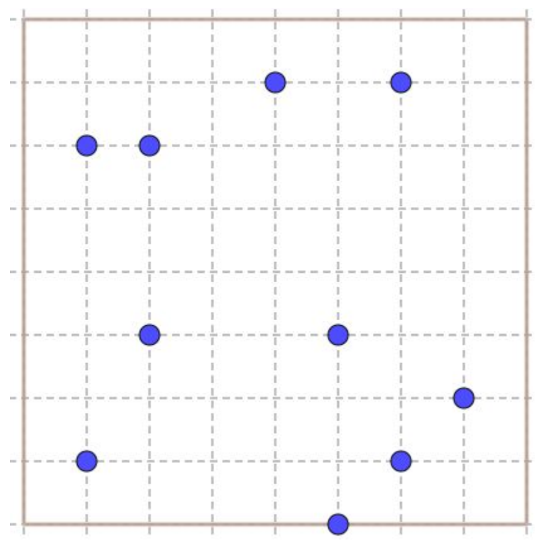
Square



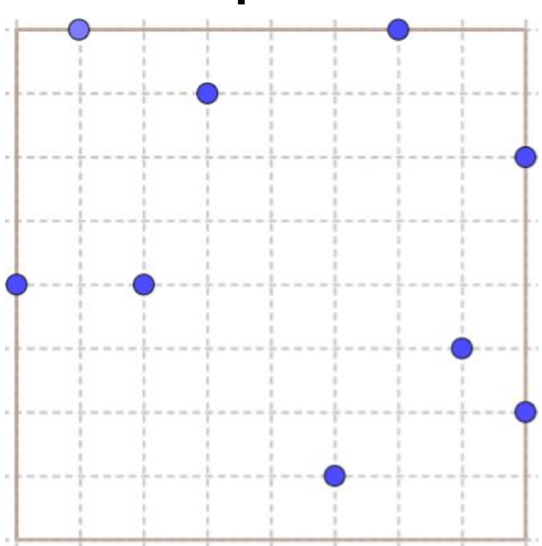
Right Scalene Triangle



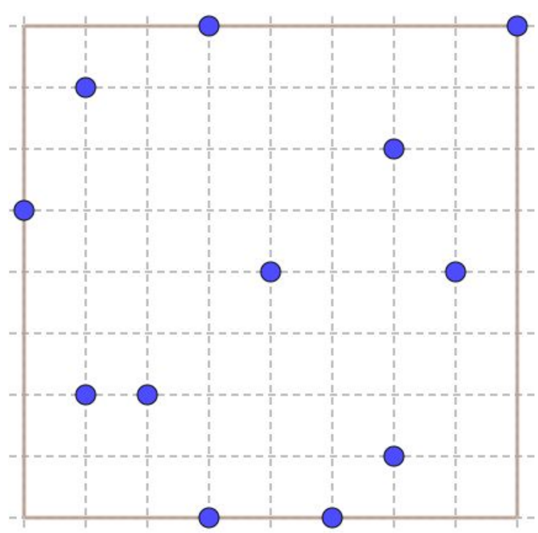
Parallelogram



Trapezoid



Right Trapezoid



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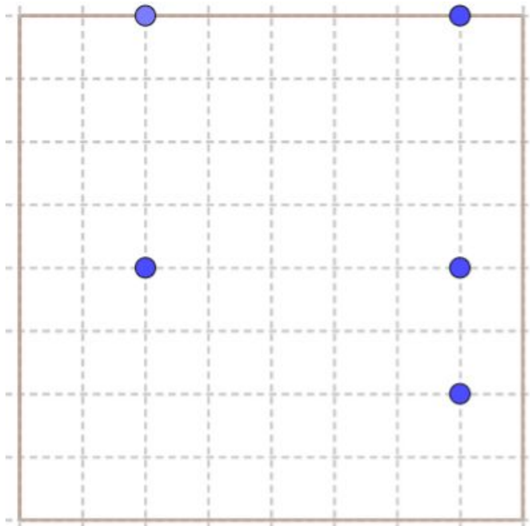
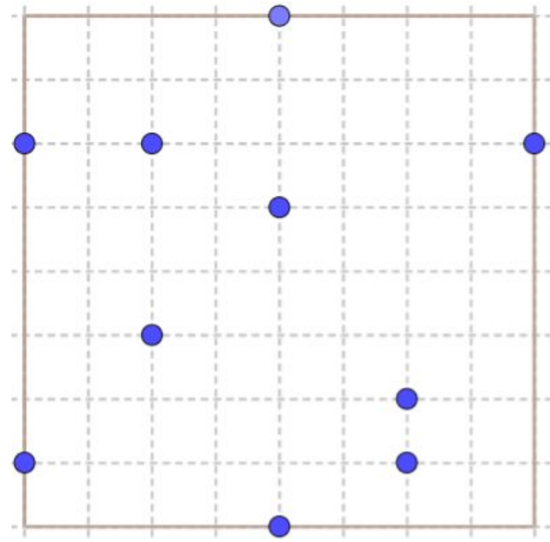
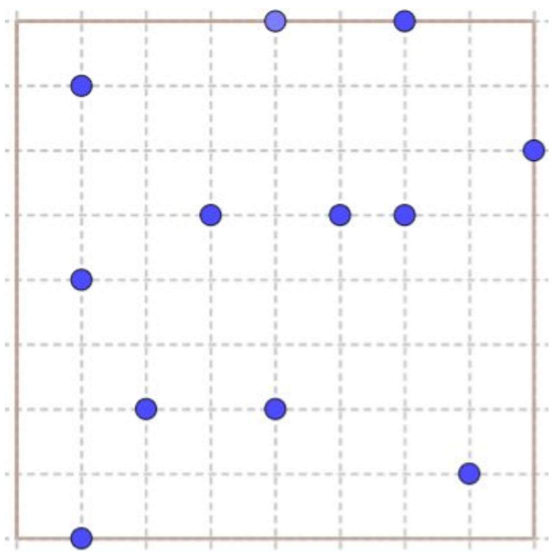
Obtuse Scalene Triangle



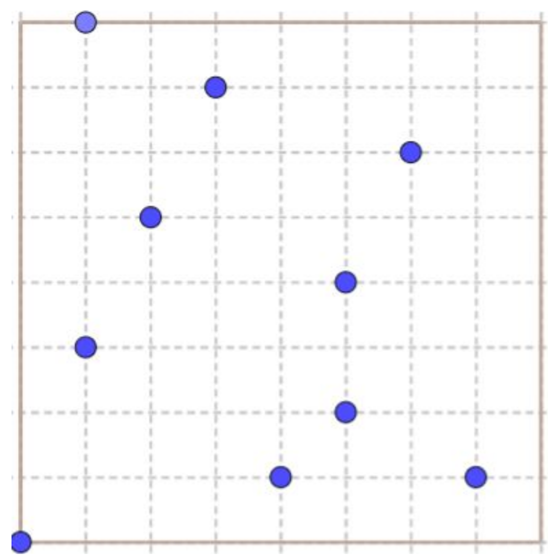
Parallelogram



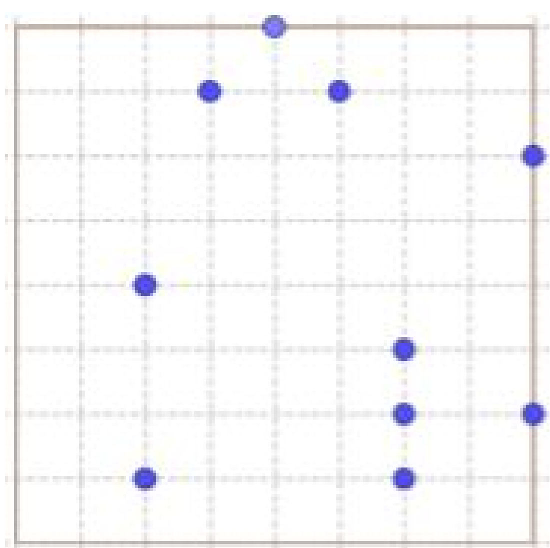
Rhombus



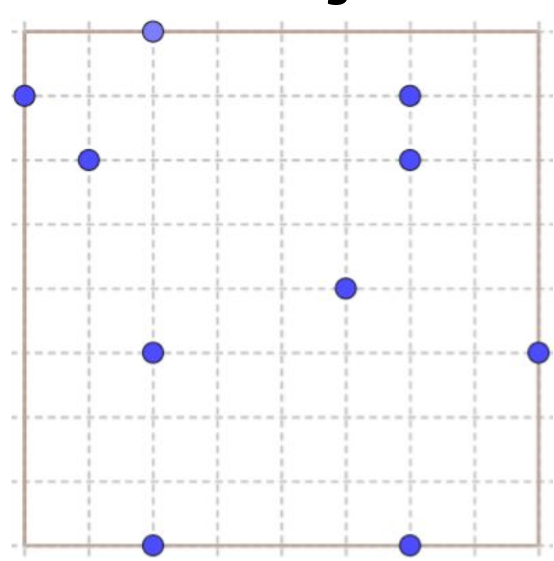
Trapezoid



Square



Rectangle



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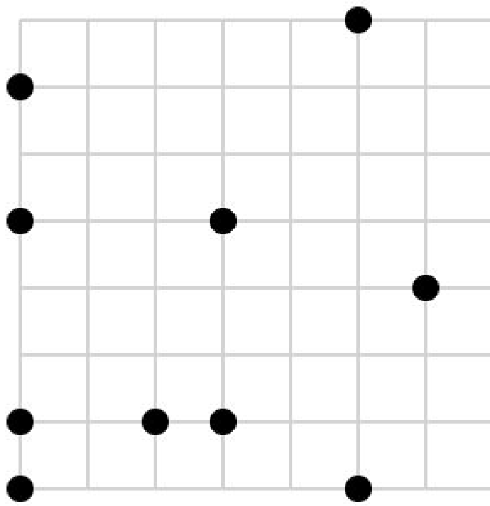
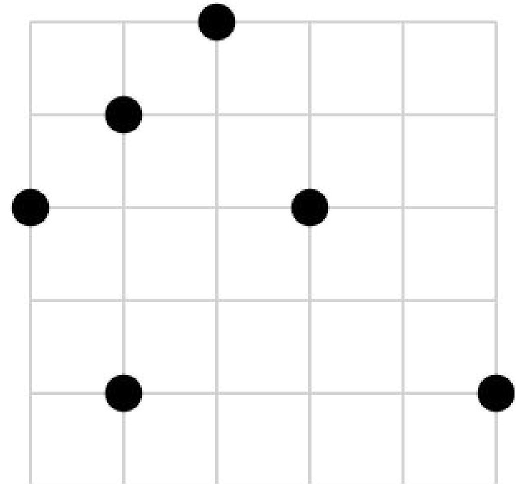
Zukei Puzzles #3

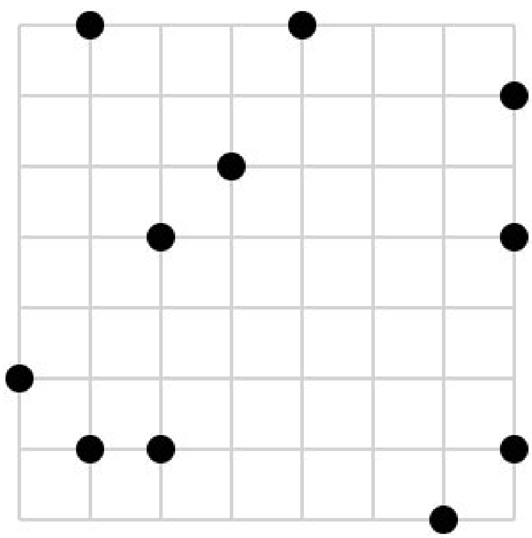
2 Squares



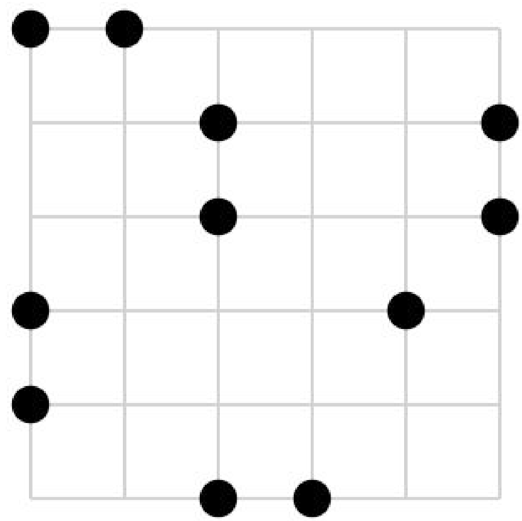
2 Right Triangles



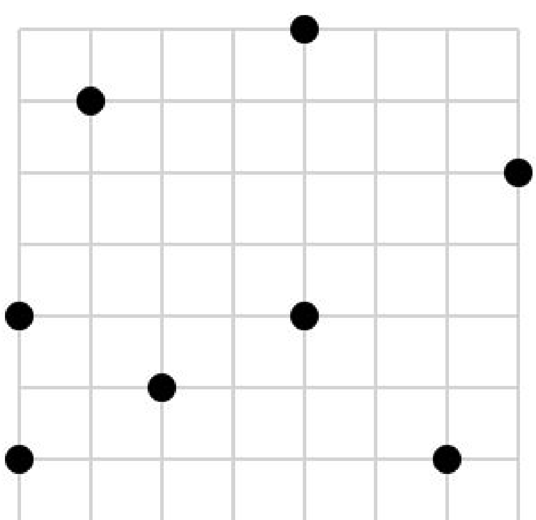
3 Parallelograms



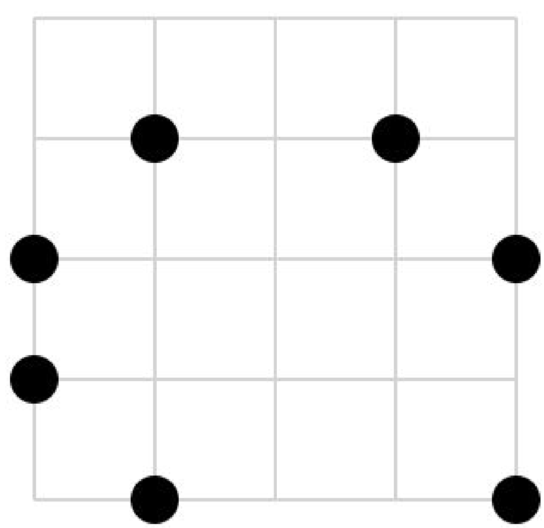
2 Rectangles



2 Trapezoids



2 Isosceles Trapezoids



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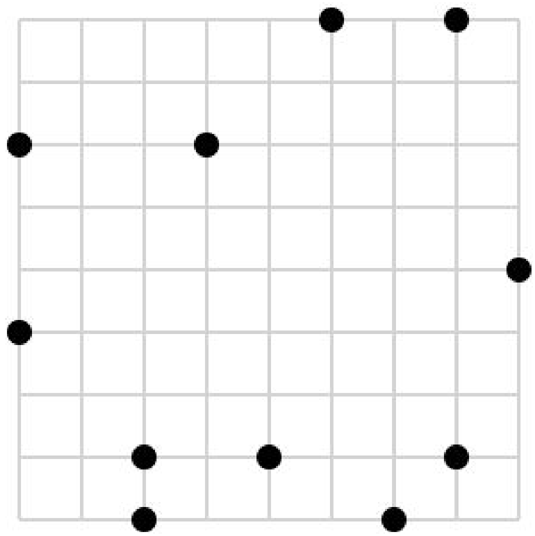
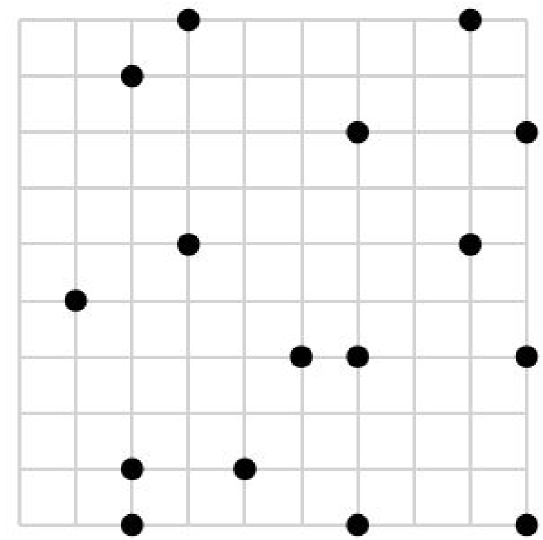
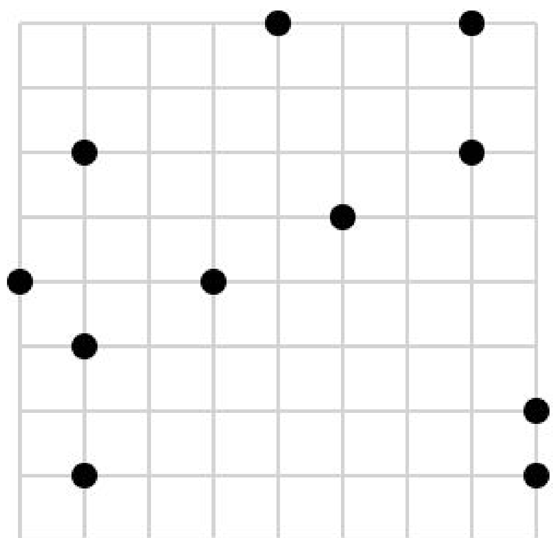
2 Parallelograms



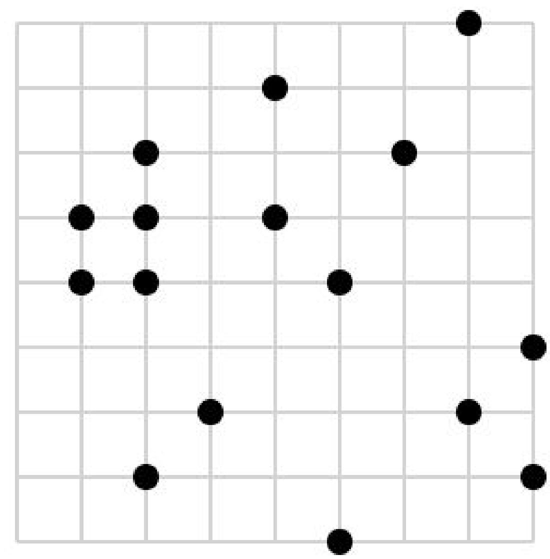
2 Squares



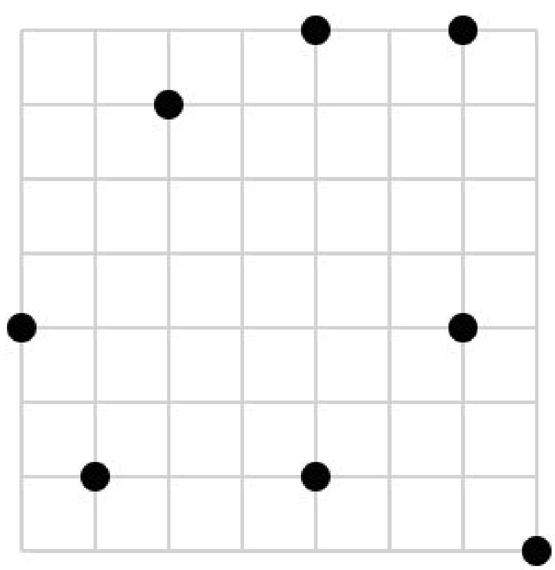
2 Isosceles Triangles



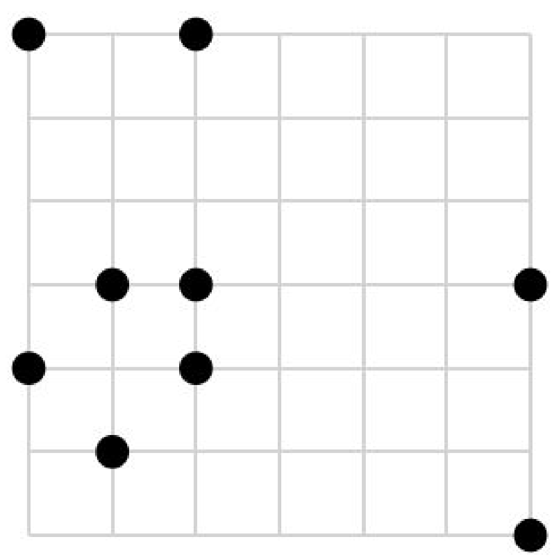
3 Rhombi



2 Right Trapezoids



3 Parallelograms



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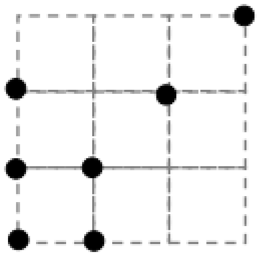
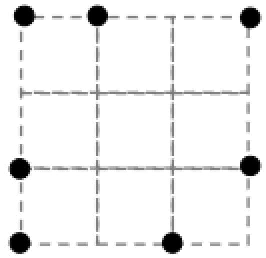
Zukei Puzzles by Naoki Inaba

(1)



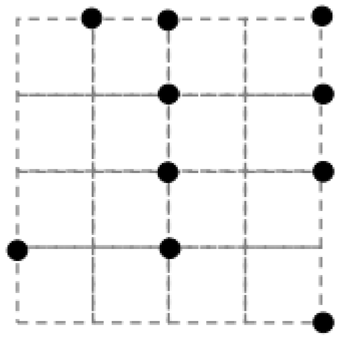
Square

(2)



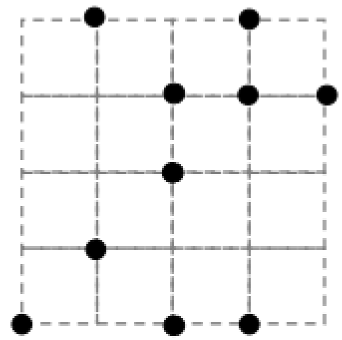
Rectangle

(3)



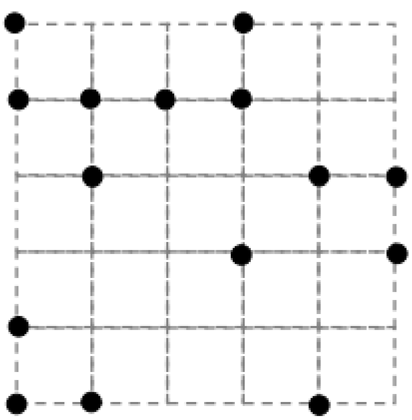
Square

(4)



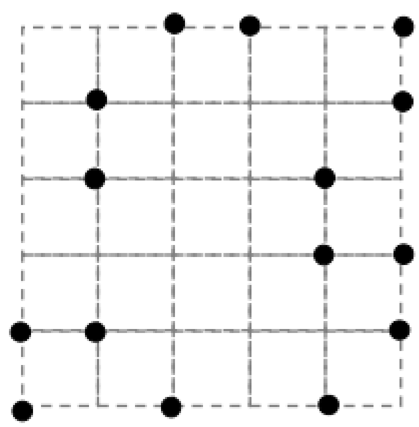
Rectangle

(5)



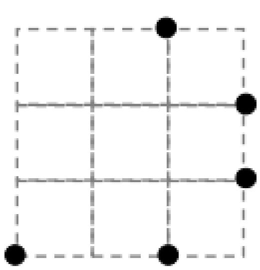
Square

(6)



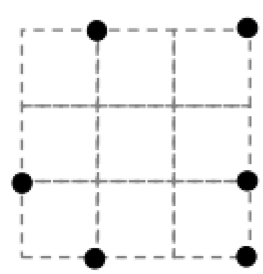
Rectangle

(7)



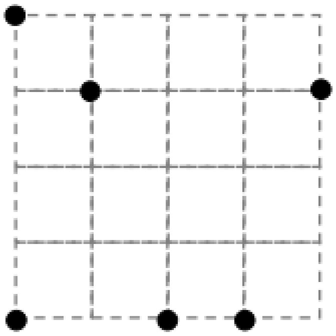
Right
Triangle

(8)



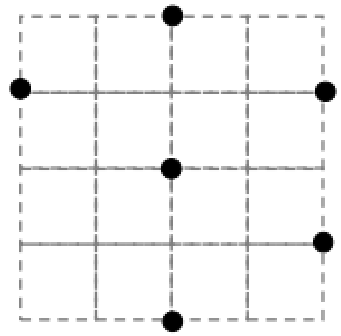
Isosceles
Right
Triangle

(9)



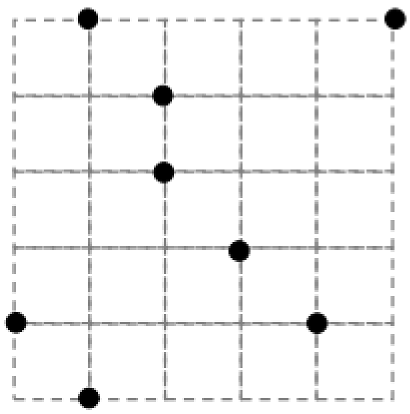
Isosceles Triangle

(10)



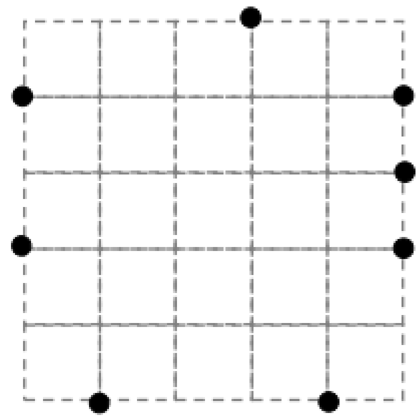
Right Triangle

(11)



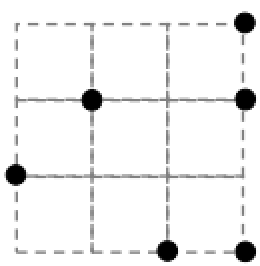
Isosceles Right Triangle

(12)



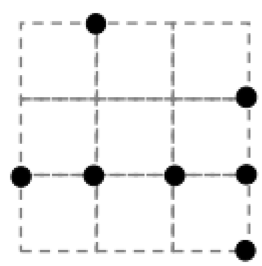
Isosceles Triangle

(13)



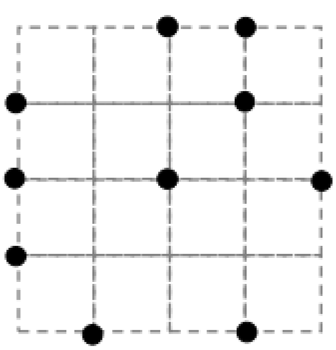
Trapezoid

(14)



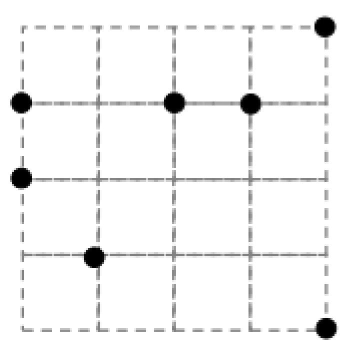
Parallelogram

(15)



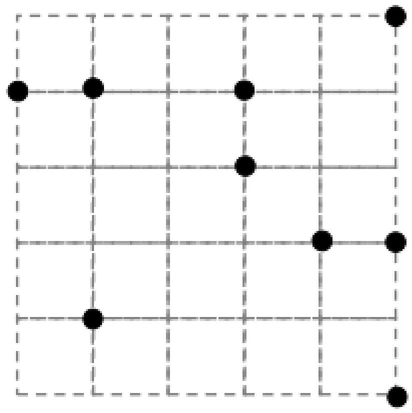
Rhombus

(16)



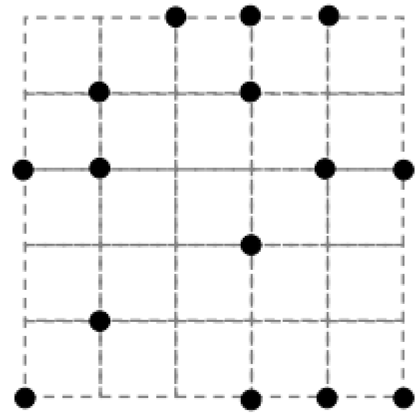
Trapezoid

(17)



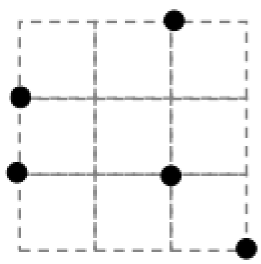
Parallelogram

(18)



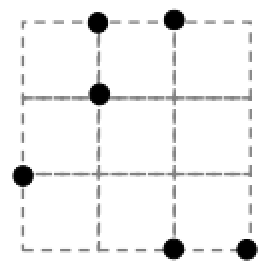
Rhombus

(19)



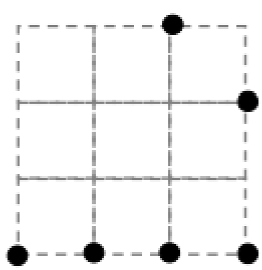
Right Triangle

(20)



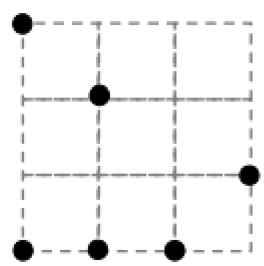
Parallelogram

(21)



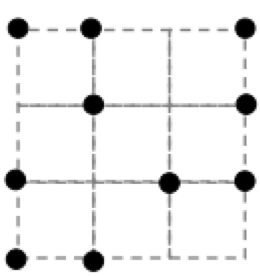
Isosceles Right Triangle

(22)



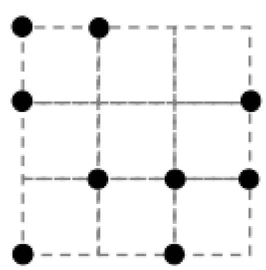
Trapezoid

(23)



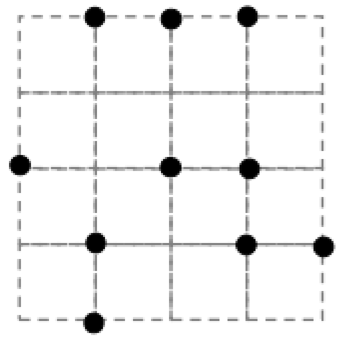
Square

(24)



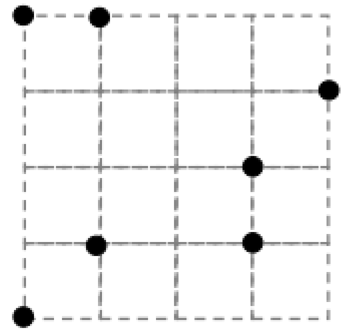
Rectangle

(25)



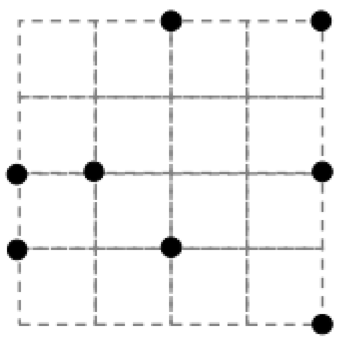
Rhombus

(26)



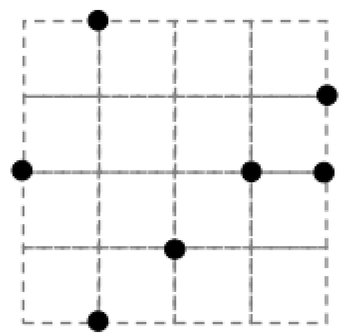
Isosceles Triangle

(27)



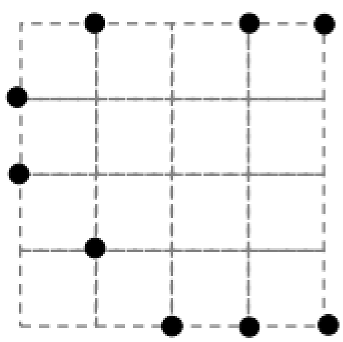
Parallelogram

(28)



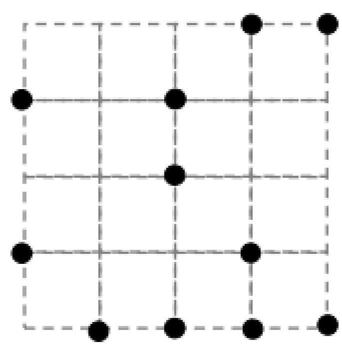
Trapezoid

(29)



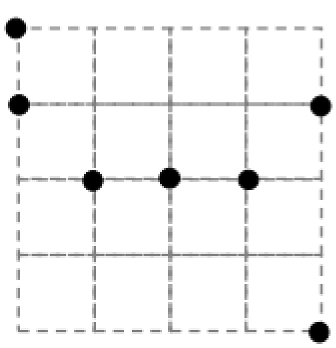
Isosceles Right Triangle

(30)



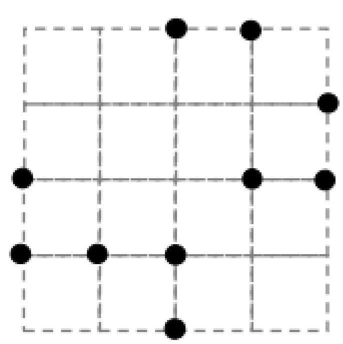
Rectangle

(31)



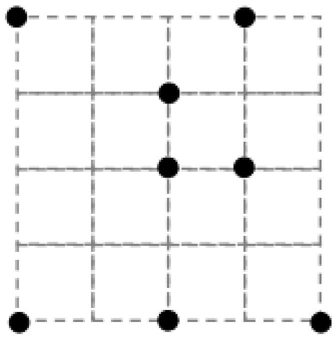
Isosceles Triangle

(32)



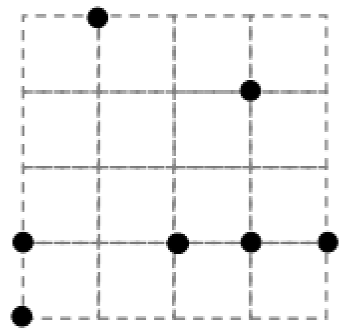
Square

(33)



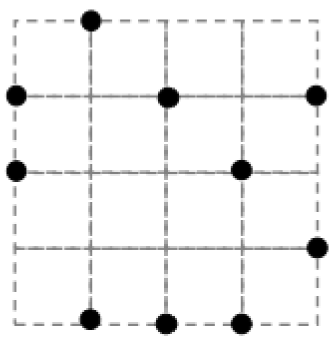
Parallelogram

(34)



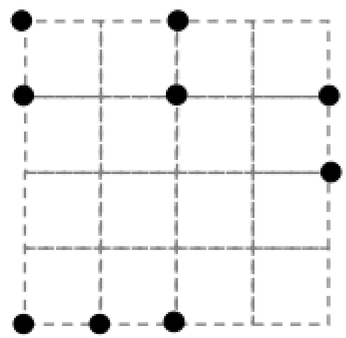
Trapezoid

(35)



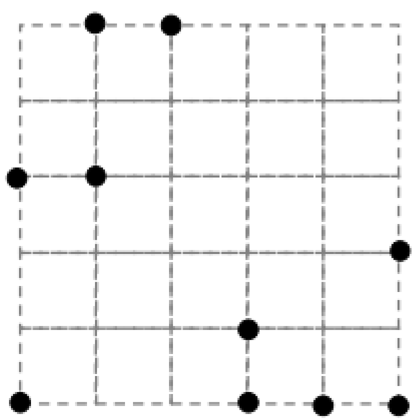
Rectangle

(36)



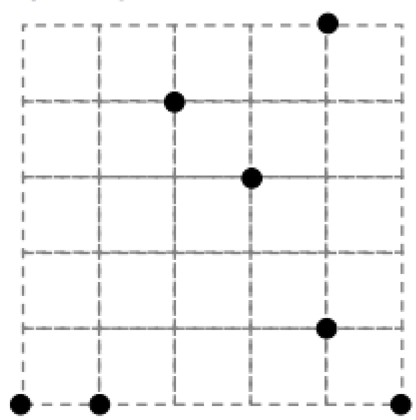
Isosceles
Right
Triangle

(37)



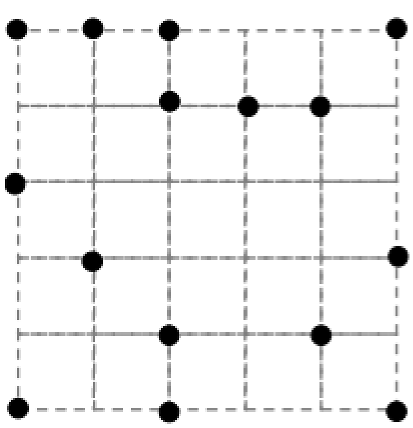
Rectangle

(38)



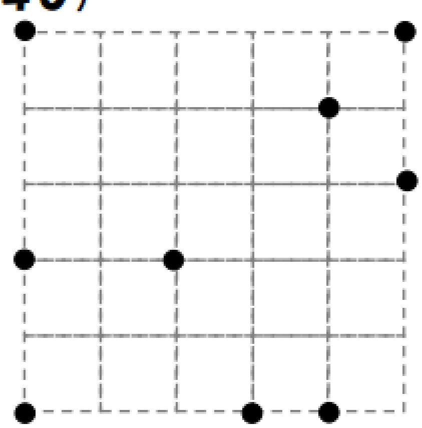
Right
Triangle

(39)



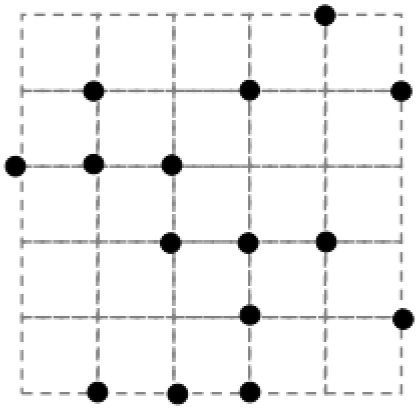
Square

(40)



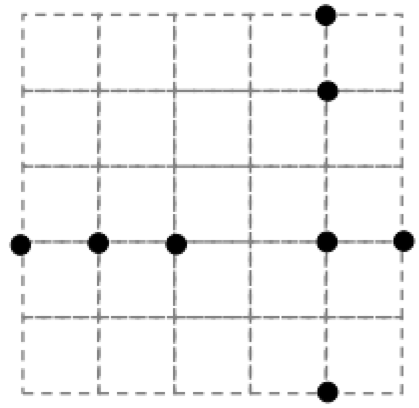
Parallelogram

(41)



Rhombus

(42)



Trapezoid