A Semi-Frivolous Attempt to Mathematize Grade Adjustments

Mike Price

April 22, 2022

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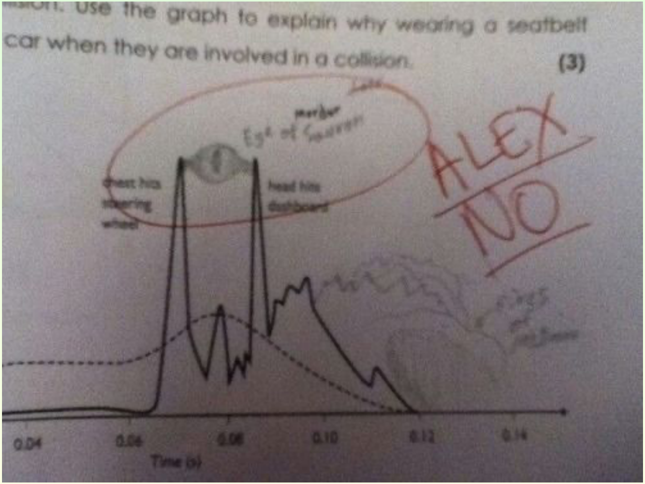

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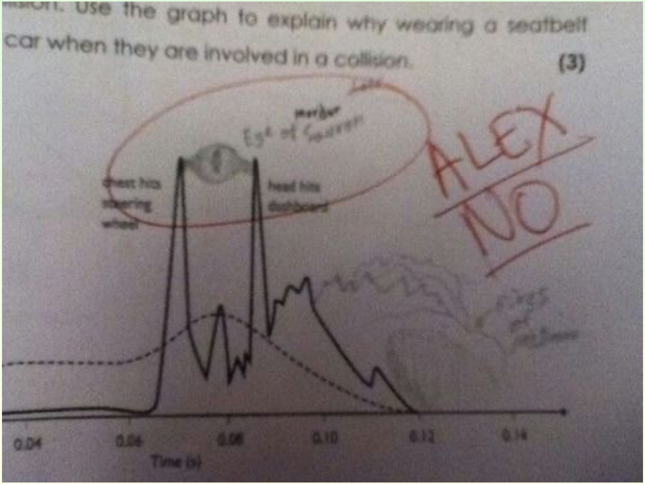

"Because otherwise the eye of Sauron will be upon you."

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- The basic mapping has no requirement that we make student grades better than their raw scores.

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Your Turn: Devise a verbal description, in terms of raw and adjusted scores, of each of the properties in the "responsible grading adjustment" function definition.

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Discuss: Is $G(r) = \cos\left(\frac{\pi}{2}r\right)$ on $[0, 1]$ a responsible grading adjustment? Nope! E.g. $G(1) = 0$, violating the second rule. (Third rule is also a problem, e.g. $G(0) > G(1)$.)

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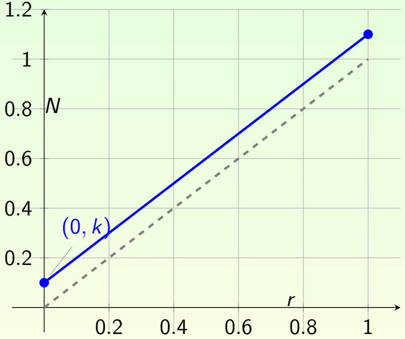

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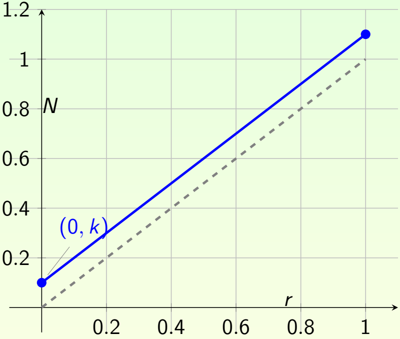

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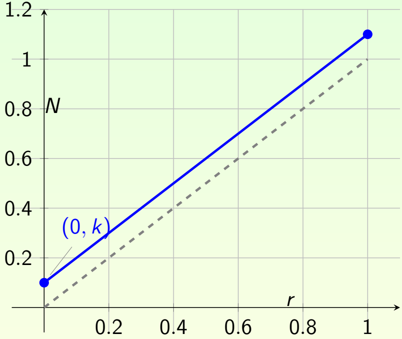

Each score adjusted exactly the same; preserves the overall spread; creates scores above 100% (good/bad?).

The "Best Student" Scaling

The "best student" scaling is the constant shift, except the new 100% is based on the highest grade that any student earned, such that the highest grade is capped at 100%. Now the adjusted grade, N, is simply the raw grade, r, plus a constant that depends on the highest grade H.

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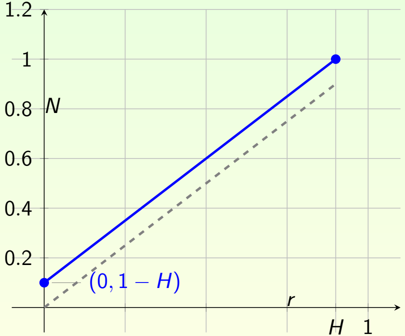

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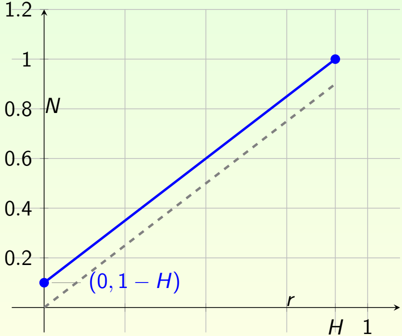

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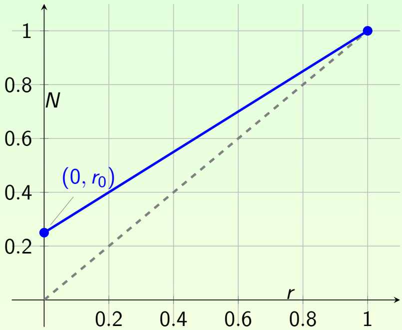

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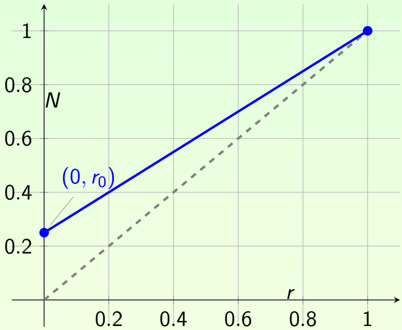

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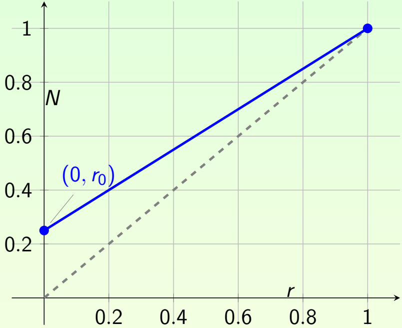

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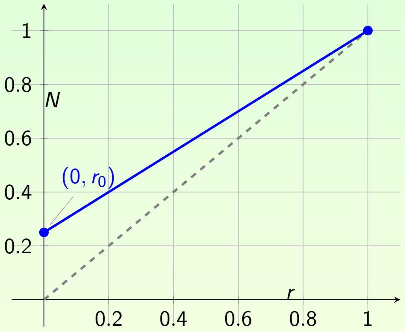

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Your Turn: Can you quickly convince yourself that condition #3 of the responsible grading adjustment definition is met for $L(r) = (1 - r_0)r + r_0$?

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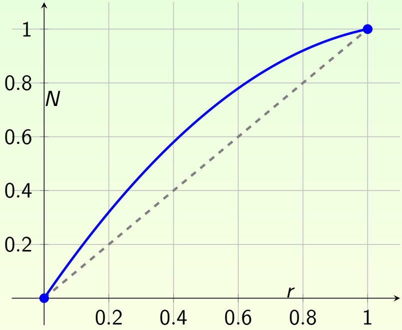

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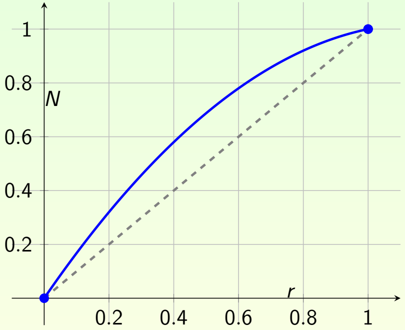

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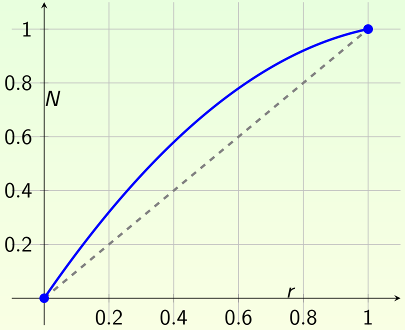

Your Turn: Why do we need the given restriction on the value of k?

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Choose a value of $0 < k < 1$, then raw scores are scaled by

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With $k = 1/2$ depicted below, the largest benefit[1] is provided to students whose raw score was 25%.

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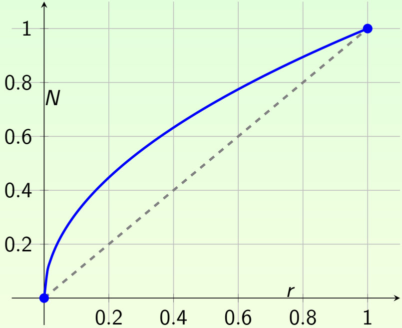

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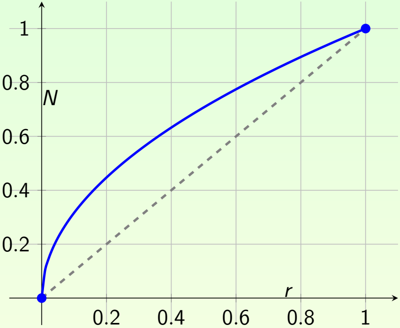

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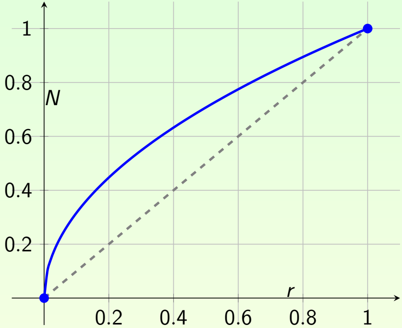

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Your Turn: Come up with your own $N = G(r)$ responsible grading adjustment as a table and share with the group!